Name: Ibe Amarachi Sheila.
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ASSIGNMENT.

1a) $\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4) \sin(\cos x)}{x - \pi/2} \right]$

SOLUTION.

Let; (Numerator).

$u = x^2 - \pi/4$; $du/dx = 2x$

$v = \sin(\cos x)$; $dv/dx = -\sin x \cos(\cos x)$

Also let;

$a = \cos x$; $da/dx = -\sin x$

$v = \sin a$; $dv/da = \cos a$

$$\frac{dv}{dx} = \frac{dv}{da} \cdot \frac{da}{dx}$$

$$\frac{dv}{dx} = \cos a \cdot (-\sin x)$$

$$\frac{dv}{dx} = -\sin x \cos a = -\sin x \cos(\cos x).$$

$$\frac{dy}{dx} = u\,\frac{dv}{dx} + v\,\frac{du}{dx}$$

$$\frac{dy}{dx} = (x^2 - \pi/4)(-\sin x \cos(\cos x)) + \sin(\cos x)(2x).$$

$$\frac{dy}{dx} = (x^2 - \pi/4)(-\cos(\cos x)\sin x) + \sin(\cos x)(2x).$$

Denominator;

Let;

$m = x - \pi/2$; $dm/dx = 1$.

$$\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)(\sin(\cos x))}{x - \pi/2} \right]$$

$$\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4) - \cos(\cos x)\sin x + \sin(\cos x)(2x)}{1} \right]$$

$$= \frac{\left((\pi/2)^2 - \pi/4\right) - \cos(\cos \pi/2)\sin(\pi/2) + \sin(\cos \pi/2)(2(\pi/2))}{1}$$

$$= \frac{(\pi^2/4 - \pi/4)(-1) + 0}{1} = -\pi^2/4 + \pi/4$$

$$= \frac{-\pi^2 + \pi}{4}$$

$$\therefore \lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)(\sin(\cos x))}{x - \pi/2}\right] = \frac{-\pi^2 + \pi}{4}$$

b) $\lim_{x \to \pi/2} \ln\left[\exp\frac{(3x^2 + 2x - 1)}{x+1}\right]$

Solution.

$$\lim_{x \to \pi/2} \ln\left[\exp\frac{(3x^2 + 2x - 1)}{x+1}\right]$$

$$= \ln\left[\exp\frac{(3(\pi/2)^2 + 2(\pi/2) - 1)}{\pi/2 + 1}\right]$$

$$= \frac{3\pi^2/4 + \pi - 1}{\pi/2 + 1}$$

$$= \frac{3\pi^2}{4} + \frac{\pi}{1} - \frac{1}{1} \div \frac{\pi}{2} + \frac{1}{1}$$

$$= \frac{3\pi^2 + 4\pi - 4}{4_2} \times \frac{2^1}{\pi + 2}$$

$$= \frac{3\pi^2 + 4\pi - 4}{2(\pi + 2)} = \frac{(3\pi - 2)(\pi + 2)}{2(\pi + 2)} = \frac{3\pi - 2}{2}$$

$$= 3/2\,\pi - 1.$$

$$\therefore \lim_{x \to \pi/2} \ln\left[\exp\frac{(3x^2 + 2x - 1)}{x+1}\right] = 3/2\,\pi - 1.$$

c) $\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right]$.

Solution.

$$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right].$$

$$= \cos\left[\sin^{-1}\frac{(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})}\right].$$

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$$= \cos\left[\sin^{-1}\sqrt{3}/2\right]$$

$$= \cos 60 = 1/2$$

$$\therefore \lim_{x\to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right] = 1/2.$$

d) $\lim_{x\to 4}\left(\frac{x^2-8x+16}{x^2-5x+4}\right)$

SOLUTION.

$$\lim_{x\to 4}\left[\frac{2x-8}{2x-5}\right]$$

$$= \frac{2(4)-8}{2(4)-5} = 0/3 = 0.$$

$$\therefore \lim_{x\to 4}\left(\frac{x^2-8x+16}{x^2-5x+4}\right) = 0.$$

2a) $\frac{2}{2\times 3} + \frac{2}{3\times 4} + \frac{2}{4\times 5} + \frac{2}{5\times 6} + \text{---}.$

SOLUTION.

$$U_n = \frac{2}{(n+1)(n+2)}$$

$$U_{n+1} = \frac{2}{(n+1+1)(n+1+2)} = \frac{2}{(n+2)(n+3)}.$$

$$\lim_{n\to\infty}\frac{U_{n+1}}{U_n} = \frac{2}{(n+2)(n+3)} \div \frac{2}{(n+1)(n+2)}$$

$$= \frac{\cancel{2}}{\cancel{(n+2)}(n+3)} \times \frac{(n+1)\cancel{(n+2)}}{\cancel{2}}$$

$$= \frac{(n+1)}{(n+3)}$$

$$\lim_{n\to\infty}\frac{n+1}{n+3} = \frac{n/n + 1/n}{n/n + 3/n}$$

$$= \frac{1 + 1/n}{1 + 3/n}$$

$n \to \infty$; $1/n \to 0$; $3/n \to 0$.

$= \frac{1+0}{1+0} = 1.$

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = 1$; $\therefore$ the series is either divergent or convergent.

$\lim_{n\to\infty} U_n \frac{2}{(n+1)(n+2)} = \frac{2/n}{n^2+3n+2}$

$= \frac{2/n^2}{1+3/n+2/n^2} = \frac{0}{1} = 0.$

$\therefore$ the series is ~~inconclusive~~ convergent since $\lim_{n\to\infty} U_n = 0.$

b) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$

SOLUTION:

$\sum_{n=1}^{\infty} 2/n^p$

$p = 2.$

since $p > 1$; $\therefore$ the series converges.

c) $U_n = \frac{1+2n^2}{1+n^2}$

SOLUTION:

$\lim_{n\to\infty} U_n$

$\lim_{n\to\infty} \frac{1+2n^2}{1+n^2} = \frac{1/n^2+2}{1/n^2+1}$

$n \to \infty$; $1/n^2 \to 0$

$= \frac{0+2}{0+1} = 2.$

since $\lim_{n\to\infty} U_n \neq 0$, $\therefore$ the series is divergent.

3.) $\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3}$

SOLUTION:

$$U_n = \frac{x^n}{(2n+1)^3}$$

$$U_{n+1} = \frac{x^{n+1}}{(2(n+1)+1))^3} = \frac{x^{n+1}}{(2n+2+1)^3} = \frac{x^{n+1}}{(2n+3)^3}$$

$$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{x^{n+1}}{(2n+3)^3} \div \frac{x^n}{(2n+1)^3}$$

$$= \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$$

$$= \frac{x(2n+1)^3}{(2n+3)^3}$$

$$\lim_{n\to\infty} \frac{x(2n+1)^3}{(2n+3)^3}$$

$$\lim_{n\to\infty} \frac{x(8n^3+12n^2+6n+1)}{(8n^3+36n^2+54n+27)}$$

$$\lim_{n\to\infty} \frac{8xn^3+12xn^2+6xn+6}{8n^3+36n^2+54n+27}$$

$$= \frac{8x + 12/n + 6x/n^2 + x/n^3}{8 + 36/n + 54/n^2 + 27/n^3}.$$

$n\to\infty$; $12x/n \to 0$, $6x/n^2 \to 0$, $x/n^3 \to 0$, $36/n \to 0$, $54/n^2 \to 0$, $27/n^3 \to 0$

$$= \frac{8x+0+0+0}{8+0+0+0} = x.$$

$$\lim_{n\to\infty} \frac{x(2n+1)^3}{(2n+3)^3} = x.$$

$-1 \le x \le 1$, $\therefore$ the series is convergent.

4.) $\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$

$\lim_{x \to 0} \left[\frac{\cos x + \sin x}{3x^2} \right]$

$\lim_{x \to 0} \left[\frac{-\sin x + \cos x}{6x} \right]$.

$\lim_{x \to 0} \left[\frac{-\cos x - \sin x}{6} \right]$

$= \frac{-\cos(0) - \sin(0)}{6} = -\frac{1-0}{6} = -\frac{1}{6}.$

$\therefore \lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right] = -\frac{1}{6}.$