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MAT NO: 14/ENG02/034

DEPT: CHEMICAL ENGINEERING

LEVEL: 500

PROCESS DYNAMICS AND CONTROL 1 (CHE 531) ASSIGNMENT 1

Given that

$$y\left(0\right)=5 and y^{'}\left(0\right)=7$$

Solve

$$\frac{d^{2}y}{dt^{2}}-3\frac{dy}{dt}+2y=2e^{3t}$$

Solution

$$y^{''}-3y^{'}+2y=2e^{3t}$$

$$(s^{2}y\left(s\right)-sy\left(0\right)-y^{'}(0))-3(sy\left(s\right)-y(0))+2y(s)= \frac{2}{s-3}$$

Inserting Initial Conditions

$$(s^{2}y\left(s\right)-5s-7)-3(sy\left(s\right)-5)+2y(s)= \frac{2}{s-3}$$

$$s^{2}y\left(s\right)-5s-7-3sy\left(s\right)+15+2y(s)= \frac{2}{s-3}$$

Collecting like terms

$$y\left(s\right)\left(s^{2}-3s+2\right)= \frac{2}{s-3}+5s-8$$

$$y\left(s\right)= \frac{2}{(s-3)(s^{2}+3s+2)}+\frac{5s-8}{(s^{2}-3s+2)}$$

$$y\left(s\right)= \frac{2+(5s-8)(s-3)}{(s-3)(s^{2}+3s+2)}$$

$$y\left(s\right)= \frac{2+5s^{2}-15s-8s+24}{(s-3)(s^{2}+3s+2)}$$

$$y\left(s\right)= \frac{5s^{2}-23s+26}{(s-3)(s^{2}+3s+2)}$$

Factorizing the second term in the denominator

$$\left(s^{2}-3s+2\right)=s^{2}-2s-s+2$$

 $=s\left(s-2\right)-1(s-2)$

Hence,$ \left(s^{2}-3s+2\right)=\left(s-2\right)(s-1)$

$$y\left(s\right)= \frac{5s^{2}-23s+26}{\left(s-1\right)\left(s-2\right)(s-3)}$$

Separating into partial fractions

$$y\left(s\right)= \frac{5s^{2}-23s+26}{\left(s-1\right)\left(s-2\right)(s-3)}=\frac{A}{(s-1)}+\frac{B}{(s-2)}+\frac{C}{(s-3)}$$

$$y\left(s\right)= \frac{5s^{2}-23s+26}{\left(s-1\right)\left(s-2\right)(s-3)}=\frac{A\left(s-2\right)\left(s-3\right)+B\left(s-1\right)\left(s-3\right)+C\left(s-1\right)(s-2)}{\left(s-1\right)\left(s-2\right)(s-3)}$$

$$y\left(s\right)= \frac{5s^{2}-23s+26}{\left(s-1\right)\left(s-2\right)(s-3)}=\frac{A\left(s^{2}-5s+6\right)+B\left(s^{2}-4s+3\right)+C\left(s^{2}-3s+2\right)}{\left(s-1\right)\left(s-2\right)(s-3)}$$

$5s^{2}-23s+26=A\left(s^{2}-5s+6\right)+B\left(s^{2}-4s+3\right)+C(s^{2}-3s+2)$

$5s^{2}-23s+26=As^{2}-5As+6A+Bs^{2}-4Bs+3B+Cs^{2}-3Cs+2C$

$5s^{2}-23s+26=\left(A+B+C\right)s^{2}+\left(-5A-4B-3C)s+(6A+3B+2C\right)$

Comparing the equations

$A+B+C=5$ (1)

$-5A-4B-3C=-23$ (2)

$6A+3B+2C=26$ (3)

Solving the above equations simultaneously by Gauss elimination method,

$\left(\begin{matrix}1&1&1\\-5&-4&-3\\6&3&2\end{matrix}\right)×\left(\begin{matrix}A\\B\\C\end{matrix}\right)=\left(\begin{matrix}5\\-23\\26\end{matrix}\right)$

For row 2 we have$ Row 2-(-5)×Row 1$

For row 3 we have $ Row 3-6×Row 1$

$$\left(\begin{matrix}1&1&1\\0&1&2\\0&-3&-4\end{matrix} \right)×\left(\begin{matrix}A\\B\\C\end{matrix}\right)=\left(\begin{matrix}5\\2\\-4\end{matrix}\right)$$

For row 3 we have $ Row 3-(-3)×Row 2$

$$\left(\begin{matrix}1&1&1\\0&1&2\\0&0&2\end{matrix} \right)×\left(\begin{matrix}A\\B\\C\end{matrix}\right)=\left(\begin{matrix}5\\2\\2\end{matrix}\right)$$

From the matrix above, we have

$A+B+C=5$ (4)

$B+2C=2$ (5)

$2C=2$ (6)

Hence, $C=1$

Substituting the value of C in (5)

$$B+2×1=2$$

$$B=2-2=0$$

Substituting the value of B and C in (4)

$$A+0+1=5$$

$$A=4$$

$$y\left(s\right)=\frac{4}{(s-1)}+\frac{1}{(s-3)}$$

Taking the Laplace inverse

$$y\left(s\right)= L^{-1}(\frac{4}{\left(s-1\right)})+L^{-1}(\frac{1}{\left(s-3\right)})$$

$$y\left(s\right)=4e^{t}+e^{3t}$$