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Question 1 : $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

We observe homogeneity

$y'' + 4y' + 5y = 6\sin\theta$

where $y = e^{k\theta}$

$y' = ke^{k\theta}$

$y'' = k^2e^{k\theta}$

$k^2e^{k\theta} + 4(ke^{k\theta}) + 5(e^{k\theta}) = 6\sin\theta$

$k^2e^{k\theta} + 4ke^{k\theta} + 5e^{k\theta} = 0$

$e^{k\theta}(k^2 + 4k + 5) = 0$

$k^2 + 4k + 5 = 0$

$k^2 + 4k = -5$

$k^2 + 4k + \left(\frac{1}{2} \times \frac{4}{1}\right)^2 = -5 + \left(\frac{1}{2} \times \frac{4}{1}\right)^2$

$k^2 + 4k + 4 = -5 + 4$

$k^2 + 2k + 2k + 4 = -1$

$(k+2)(k+2) = -1$

$(k+2)^2 = -1$

$k + 2 = \sqrt{-1} \Rightarrow \pm i$

$k_1 = -2 + i$ and $k_2 = -2 - i$

$y_h = C_1y_1 + C_2y_2$

$y_1 = e^{k\theta} = e^{(-2+i)x} = e^{-2x} \cdot e^{ix}$

$y_2 = e^{k_2\theta} = e^{(-2-i)x} = e^{-2x} \cdot e^{ix}$

$y_h = C_1e^{-2x} \cdot e^{ix} + C_2e^{-2x} \cdot e^{ix}$

$y_h = e^{-2x}(C_1e^{ix} + C_2e^{-ix})$

$y_h = e^{-2\theta}(C_1\cos\theta + C_2\sin\theta)$

Assume $y_p = A\cos\theta + B\sin\theta$

$y_p' = -A\sin\theta + B\cos\theta$

$y_p'' = -A\cos\theta - B\sin\theta$

Recall $y'' + 4y' + 5y = 6\sin\theta$

$-A\cos\theta - B\sin\theta + 4(-A\sin\theta + B\cos\theta) + 5(A\cos\theta + B\sin\theta) = 6\sin\theta$

$-A\cos\theta - B\sin\theta - 4A\sin\theta + 4B\cos\theta + 5A\cos\theta + 5B\sin\theta = 6\sin\theta$

$4A\cos\theta + 4B\sin\theta - 4A\sin\theta + 4B\cos\theta = 6\sin\theta$

$(4A + 4B)\cos\theta + (4B - 4A)\sin\theta = 6\sin\theta$

$(4A + 4B)\cos\theta + (4B - 4A)\sin\theta = 6\sin\theta$

$4B - 4A = 6 \quad --- (1)$

$4A + 4B = 0 \quad --- (2)$

$4B - 4A = 6 \quad --- (1)$

$4B + 4A = 0 \quad --- (2)$

$\frac{8B}{8} = \frac{6}{8} \quad \frac{3}{4}$

$B = 3/4$

$4(3/4) - 4A = 6$

$12/4 - 4A = 6$

$4A = 6 - 12/4$

$\frac{-4A}{-4} = \frac{3}{-4}$

$A = -3/4$

$y_p = -3/4\cos\theta + 3/4\sin\theta$

$y = y_h + y_p$

$y = e^{-2\theta}(C_1\cos\theta + C_2\sin\theta) - 3/4\cos\theta + 3/4\sin\theta$

iii) at $\theta = \infty$ and $\frac{dy}{d\theta} = 0$

$\frac{dy}{d\theta} = e^{-2\theta}[-C_1\sin\theta + C_2\sin\theta] + [C_1\cos\theta + C_2\sin\theta]-2e^{-2t} + 3/4\sin\theta + 3/4\cos\theta$

at $\theta = \infty$ and $dy/d\theta = 0$

$0 = 3/4\sin\theta + 3/4\cos\theta$

$-3/4\cos\theta = 3/4\sin\theta$

$-\cos\theta = \sin\theta$

divide both side by $-\cos\theta$

$$\frac{\sin\theta}{-\cos\theta} = 1$$

$$-\tan\theta = 1$$

$$\tan\theta = -1$$

$$\theta = \tan^{-1}(-1)$$

$$\theta = -45^\circ$$

Question 2b

$$EI\, \frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2$$

Solution

$$EIm^2 = 0$$

$$m^2 = 0$$

$$m = \pm\sqrt{0}$$

$$m = \pm 0$$

$$y = e^{0x}(A+Bx)$$

$$y = A + Bx \quad --- (1)$$

Assume $y_p = Px^2 + Qx^3 + Rx^4$

$$y_p' = 2Px + 3Qx^2 + 4Rx^3$$

$$y_p'' = 2P + 6Qx + 12Rx^2$$

$$EI[2P + 6Qx + 12Rx^2] = \frac{w}{2}(L-x)^2$$

$$2PEI + 6QEIx + 12REIx^2 = \frac{w}{2}(L^2 - 2Lx + x^2)$$

$$4PEI + 12QEIx + 24REIx^2 = w(L^2 - 2Lx + x^2)$$

$$24REI = w$$

$$R = \frac{w}{24EI} \quad --- (2)$$

$$12QEI = -2wL$$

$$Q = \frac{-2wL}{12EI} = \frac{-wL}{6EI} \quad --- (3)$$

$$4PEI = wL^2$$

$$P = \frac{wL^2}{4EI}$$

$$y = \left[\frac{wL^2}{4EI}\right]x^2 - \left[\frac{wL}{6EI}\right]x^3 + \left[\frac{w}{24EI}\right]x^4$$

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$$y = \frac{wL^2 x^2}{4EI} - \frac{wL x^3}{6EI} + \frac{w x^4}{24EI}$$

$$y = \frac{w x^2}{24EI}\left[x^2 - 4Lx + 6L^3\right]$$

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