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1a) $\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2}\right]$

Soln - Numerator

Let $u = x^2 - \pi/4$; $du/dx = 2x$

$v = \sin(\cos x$; $dv/dx = -\sin x \cos(\cos x)$

Also let

$a = \cos x$; $du/dx = -\sin x$

$v = \sin a$; $dv/da = \cos a$

$\frac{dv}{dx} = \frac{dv}{da} \times \frac{da}{dx}$

$dv/dx = \cos a \times (-\sin x)$

$dv/dx = -\sin x \cos a = -\sin x \cos(\cos x)$

$dy/dx = u\, dv/dx + v\, du/dx$

$dy/dx = (x^2 - \pi/4)(-\sin x \cos(\cos x)) + \sin(\cos x)(2x)$

$\frac{dy}{dx} = (x^2 - \pi/4)(-\cos(\cos x)\sin x) + \sin(\cos x)(2x)$

Denominator

Let $m = x - \pi/2$ $dm/dx = 1$

$\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)(\sin(\cos x))}{x - \pi/2}\right]$

$\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4) - \cos(\cos x)\sin x + \sin(\cos x)(2x)}{1}\right]$

$= \frac{(\pi/2)^2 - \pi/4 - \cos(\cos \pi/2)\sin(\pi/2) + \sin(\cos \pi/2)(2(\pi/2))}{1}$

$= \frac{(\pi^2/4 - \pi/4)(-1) + 0}{1} = -\frac{\pi^2}{4} + \frac{\pi}{4}$

$= \frac{-\pi^2 + \pi}{4}$

$\therefore \lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2}\right]$ is $\frac{-\pi^2 + \pi}{4}$

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11.) $\lim_{x \to \pi/2} \ln\left(\exp\frac{(3x^2+2x-1)}{x+1}\right)$

Soln

$\lim_{x \to \pi/2} \ln\left(\exp\frac{(3x^2+2x-1)}{x+1}\right)$

$= \ln\left(\exp\frac{(3(\pi/2)^2+2(\pi/2)-1)}{\pi/2+1}\right)$

$= \ln\left(\exp\frac{3\pi^2/4+\pi-1}{\pi/2+1}\right)$

$= \frac{3\pi^2}{4} + \frac{\pi}{1} - \frac{1}{1} \div \frac{\pi}{2} + 1$

$= \frac{3\pi^2+4\pi-4}{4} \div \frac{\pi+2}{2}$

$= \frac{3\pi^2+4\pi-4}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{\pi+2} = \frac{3\pi^2+4\pi-4}{2(\pi+2)}$

$= \frac{(3\pi-2)(\pi+2)}{2(\pi+2)} = \frac{3\pi-2}{2}$

$\therefore \lim_{x \to \pi/2} \ln\left(\exp\frac{(3x^2+2x-1)}{x+1}\right) = \frac{3}{2}\pi - 1$

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c) $\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right]$

Soln

$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right]$

$= \cos\left[\sin^{-1}\frac{(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})}\right]$

$= \cos\left[\sin^{-1}\frac{\sqrt{3}}{2}\right]$

$= \cos 60$

$= \frac{1}{2}$

d) $\lim_{x \to 4} \left(\frac{x^2-8x+16}{x^2-5x+4}\right)$

Soln

$\lim_{x \to 4} \left(\frac{2x-8}{2x-5}\right)$

$= \frac{2(4)-8}{2(4)-5} = \frac{0}{3} = 0$

$\lim_{x \to 4} \left(\frac{x^2-8x+16}{x^2-5x+4}\right) = 0$

2a) $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \ldots$

$$U_n = \frac{2}{(n+1)(n+2)}$$

$$U_{n+1} = \frac{2}{(n+1+1)(n+1+2)}$$

$$U_{n+1} = \frac{2}{(n+2)(n+3)}$$

$$\frac{U_{n+1}}{U_n} = \frac{2}{(n+2)(n+3)} \div \frac{2}{(n+1)(n+2)}$$

$$= \frac{\cancel{2}}{\cancel{(n+2)}(n+3)} \times \frac{(n+1)\cancel{(n+2)}}{\cancel{2}}$$

$$=$$

$$\lim_{x\to\infty} \div \frac{U_{n+1}}{U_n} = \frac{n+1}{n+3} = \frac{n/n + 1/n}{n/n + 3/n} = \frac{1+0}{1+0} = 1$$

It is inconclusive Testing further

$$U_n = \frac{2}{(n+1)(n+2)}$$

$$\lim_{n\to\infty} U_n = \frac{2/n}{(n/n + 1/n)(n/n + 2/n)}$$

$$= \frac{0}{(1+0)(1+0)}$$

$$= \frac{0}{1\times1} = 0$$

If $\lim_{n\to\infty} U_n = 0$ it is convergent

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2b) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$

$u_n = \frac{2}{n^2}$

$u_{n+1} = \frac{2}{(n+1)^2}$

$\frac{u_{n+1}}{u_n} = \frac{2}{(n+1)^2} \div \frac{2}{n^2}$

$= \frac{\cancel{2}}{(n+1)^2} \times \frac{n^2}{\cancel{2}}$

$= \frac{n}{(n+1)^2}$

$= \frac{n^2}{n^2+2n+1}$

$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \frac{n^2/n^2}{n^2/n^2 + 2n/n + 1/n^2}$

$= \frac{1}{1+0+0}$

$= 1$

Inconclusive $\rightarrow$ Test further

$u_n = \frac{2}{n^2}$

$\lim_{n\to\infty} u_n = \frac{2/n^2}{n^2/n^2}$

$= \frac{0}{1}$

$= 0$

the series is convergent

2c

$$\lim_{n\to\infty} U_n = \frac{1+2n^2}{1+n^2}$$

$$U_n = \frac{\frac{1}{n^2} + \frac{2n^2}{n^2}}{\frac{1}{n^2} + \frac{n^2}{n^2}}$$

$$= \frac{\frac{1}{n^2} + 2}{\frac{1}{n^2} + 1}$$

as $n \to \infty$, $\frac{1}{n^2} = 0$

$$U_n = \frac{0+2}{0+1}$$

$$U_n = 2$$

$\therefore$ The series is divergent Singe $U_n \neq 0$

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3) $\frac{x}{27} + \frac{x^2}{125} + \ldots + \frac{x^n}{(2n+1)^3}$

$U_n = \frac{x^n}{(2n+1)^3}$

$U_{n+1} = \frac{x^{n+1}}{(2n+1)^3+1}$

$U_{n+1} = \frac{x^{n+1}}{(2n+1)^3+1}$

$U_{n+1} = \frac{x^{n+1}}{(2(n+2)+1)}$

$U_{n+1} = \frac{x^{n+1}}{(2n+3)^3}$

$U_{n+1} = \frac{x^{n+1}}{8n^3+36n^2+54n+27} \div \frac{x^n}{(2n+1)^3}$

$= \frac{x^{n+1}}{8n^3+36n^2+54n+27} \times \frac{(2n+1)^3}{x^n}$

$= \frac{x^n \cdot x^1}{8n^3+36n^2+54n+27} \times \frac{8n^3+12n^2+6n+1}{x^n} = x \times \frac{8n^3+12n^2+6n+1}{8n^3+36n^2+54n+27}$

$= x\left[\frac{8n^3+12n^2+16n+1}{8n^3+36n^2+54n+27}\right] = \underset{n\to\infty}{x}\left[\frac{8+12/n+6/n^2}{8+36/n+54/n^2+27/n^3}\right]$

$= x\left[\frac{8+0+0}{8+0+0+0}\right]$

$\frac{U_{n+1}}{U_n} = x$

$= -1 \leq x < 1$ (it converges)

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4) $\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$

$= \frac{\sin 0 - \cos 0}{0^3}$

$= \frac{0-1}{0} = 0$ (undefined)

$\lim_{x \to 0} \left[\frac{\cos x + \sin x}{3x^2} \right]$

$\lim_{x \to 0} = \frac{1+0}{0}$ = (undefined

$\lim_{x \to 0} \left[\frac{-\sin x + \cos x}{6x} \right]$

$= \frac{0+1}{0}$ (undefined

$\lim_{x \to 0} \left\{ \frac{-\cos x - \sin x}{6} \right\}$

$= \frac{-1-0}{6} = \frac{-1}{6}$