Name: Mbong Akanyene A.
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Dept: Petroleum Engineering.

ENG 381

1. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$ ——— (1)

$\cancel{y'' - y' - 2y = 8}$
$\cancel{y = e^{kx}}$

$m^2 - m - 2 = 0$
$m^2 - 2m + m - 2 = 0$
$(m^2 - 2m) + (m - 2) = 0$
$m(m-2) + 1(m-2) = 0$
$(m+1)(m-2)$
$m = 2$ or $m = -1$
Recall $y = Ae^{2x} + Be^{-x}$
$y = Ae^{+2x} + Be^{-x}$
let $y = c$
$\frac{dy}{dx} = 0$ $\quad \frac{d^2y}{dx^2} = 0$
$-2c = 8$
$c = \frac{8}{-2}$ $\quad c = -4$
PI $y = -4$
$GS = CF + PI$
$y = Ae^{2x} + Be^{-x} - 4$

2. $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$ ——— (1)

$f(x) = 0$
$\frac{d^2y}{dx^2} - 4y = 0$
$\frac{d^2y}{dx^2} = -4y$
$\frac{d^2y}{dx^2} - 4y = 0$
$m^2 - 4 = 0$
$m^2 = 4$
$m = \sqrt{4}$
$m = \pm 2$
$y = Ae^{2x} + Be^{-2x}$ – CF
Getting PI
let $y = Ce^{3x}$
$\frac{dy}{dx} = 3Ce^{3x}$
$\frac{d^2y}{dx^2} = 9Ce^{3x}$
putting the values into — (1)
$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$
$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$
$Ce^{3x}(9-4) = 10e^{3x}$
$5C\cancel{e^{3x}} = 10\cancel{e^{3x}}$
$5C = 10$
$C = \frac{10}{5}$
$C = 2$
PI
$y = 2e^{3x}$
$GS = CF + PI$
$y = Ae^{2x} + Be^{-2x} + 2e^{3x}$

3. $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = e^{-2x}$ ——— (1)

When $f(x) = 0$
$\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = 0$
$m^2 + 2m + 1 = 0$
$(m^2 + m)(m + 1) = 0$
$m(m+1) + 1(m+1) = 0$
$(m+1)(m+1) = 0$

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$m = -1$ twice

$y = e^{-x}(A + Bx)$ - CF

let $y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

Putting values in — (1.)

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x}(4-4+1) = e^{-2x}$

$C\cancel{e^{-2x}} = \cancel{e^{-2x}}$

$C = 1$

$PI = y = e^{-2x}$

$GS = CF + PI$

$y = e^{-x}(A + Bx) + e^{-2x}$

(4.) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$ —(1)

$f(x) = 0$

$\frac{d^2y}{dx^2} + 25y = 0$

$m^2 + 25 = 0$

$m^2 = -25$

$m = \sqrt{-25}$

$m = \pm 5i$

$y = C\cos 5x + D\sin 5x$ - CF

To get PI

$y = Cx^2 + Dx + Ex + F + D$

$\frac{dy}{dx} = 2Cx + D + C$

$\frac{d^2y}{dx^2} = 2C$

Putting values into —(1.)

$2C + 25i\ Cx^2 + Dx + E + Cx + D = 5x^2 + x$

$x + 25\ Cx^2 + 25Dx + 25E + 25Cx + 25D$

$= 5x^2 + x$

$2C = 5x^2 + x$

$25Cx^2 = 5x^2 + x$

$25Dx = 5x^2 + x$

$25E = 5x^2 + x$

$25Cx = 5x^2 + x$

$25D = 5x^2 + x$

$y = Cx^2 + Dx + Ex + F$

$\frac{dy}{dx} = 2Cx + \cancel{D + E}$

$\frac{d^2y}{dx^2} = 2C$

$2C + 25Cx^2 + 25Dx + 25Ex = 5x^2 + x$

$2C = 5$

$25C = 5$

$C = 5/25$

$D = 1/25$

$E = 1/25$

$\cancel{E = 2 + 2}$ $\cancel{+ 2C}$

$2C + 25E = 0$

$2(\frac{1}{5}) + 25E = 0$

$25E = -\frac{2}{5}$

$E = \frac{-2}{125}$

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(5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$f(x) = 0$

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

$m^2 - 2m + y = 0$

$m^2 - m - m + 1 = 0$

$(m^2 - m)(-m + 1) = 0$

$m(m-1) - 1(m-1) = 0$

$(m-1)(m-1)$

$m_1 = 1$ twice

$y = e^x(A + Bx)$ — CF

To get PI

$y = C\cos x + D\sin x$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

Sub into —(1)

$-C\cos x - D\sin x - 2(C\sin x + D\cos x)$

$+ C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x$

$+ C\cos x + D\sin x = 4\sin x$

$\cos x(-C - 2D + C) + \sin x(-D + 2C + D)$

$= 4\sin x$

$\cos x(-2D) + \sin x(2C) = 4\sin x$

$-2D = 0$

$D = 0$

$2C = 4$

$C = 2$

$PI = y = 2\cos x + D\sin x$

$GS = PI + CF$

$y = e^x(A + Bx) + 2\cos x$

(6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$ —(1)

$x = 0 \quad y = 1 \quad \frac{dy}{dx} = -2$

$f(x) = 0$

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$

$m^2 + 4m + 5 = 0$

Using general quadratic formulae

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2 \times 1}$

$\frac{-4 \pm \sqrt{-4}}{2}$

$\frac{-4}{2} \pm \frac{\sqrt{-4}}{2}$

$\frac{-4}{2} \pm \frac{+2i}{2}$

$-2 \pm i$

$m = -2 + i$ or $-2 - i$

$y = e^{-2x}(C\cos x + D\sin x)$ — CF

Let $y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

Put values into —(1)

~~$4Ce^{-2x} + 4$~~

$y = Cxe^{-2x}$

$\frac{d^2y}{dx^2} = -2Cx(-2e^{-2x}) + e^{-2x}(-2C)$

$+ Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Cxe^{-2x} - 4Ce^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} - 8Cxe^{-2x} + 4Ce^{-2x}$
$+ 5Cxe^{-2x} = 2e^{-2x}$
$+ \cancel{4Cxe^{-2x}} - 8Cxe^{-2x} + 5Cxe^{-2x}$
$- \cancel{4Ce^{-2x}} + \cancel{4Ce^{-x}} = 2e^{-2x}$
$(Cx - 8Cx + 5Cx) = 2e^{-2x}$
$Cx = 2$
$Cx = 2$
PI $y = 2e^{-2x}$
$y = e^{-2x}(C\cos x + D\sin x) + 2e^{-2x}$
at $x=0$, $y=1$
$e^{-2(0)}(C\cos(0) + D\sin 0) + 2e^{-2(0)}$
$y = 1(C+0) + 2$
$y = C + 2$
$C = -1$
$\frac{dy}{dx} = e^{-2x}(-C\sin x + D\cos x) +$
$(C\cos x + D\sin x) - 2e^{-2x}$
$+ 4e^{-2x}$
at $x=0$ $y=1$ $\frac{dy}{dx} = -2$
$-2 = D - 2C - 4$
$-2 + 4 = D - 2C$
$2 = D + 2$
$D = 0$
$y = e^{-2x}(-\cos x + 0) + 2e^{-2x}$
$y = e^{-2x}(-\cos x) + 2e^{-2x}$
$y = e^{-2x}(2 - \cos x)$

(7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$ —(1)
let $f(x) = 0$
$3m^2 - 2m - 1 = 0$
$3m^2 - 3m + m - 1 = 0$
$(3m^2 - 3m) + (m-1) = 0$
$3m(m-1) + (m-1) = 0$
$m = 1$ $m = -1/3$

$y = Ae^{x} + Be^{-1/3x}$
find PI
$y = Cx + D$
$\frac{dy}{dx} = C$
$\frac{d^2y}{dx^2} = 0$
sub into —(1)
$0 - 2C - Cx - D = 2x - 3$
$-Cx - 2C - D = 2x - 3$
$-Cx = 2x$
$C = -2$
$-2C - D = -3$
$-4C - D = -3$
$+D = +7$
$D = 7$
PI $= -2x + 7$
$y = Ae^{x} + Be^{-1/3} - 2x + 7$

(8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$
$f(x) = 0$
$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$
$m^2 - 6m + 8 = 0$
$m^2 - 4m - 2m + 8 = 0$
$(m-4)(m-2)$
$m = 4$, $m = 2$
$y = Ae^{4x} + Be^{2x}$

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get PI

$y = Ce^{4x}$

Since CF contains the same e^{4x} then we add Cx

$y = Cxe^{4x}$

$\frac{dy}{dx} = Cx(4e^{4x}) + e^{4x}(C)$

$\frac{d^2y}{dx^2} = Cx(16e^{4x}) + 4e^{4x}(C) + Ce^{4x}$

$y = Cxe^{4x}$

$\frac{dy}{dx} = 4Cxe^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = 4Cx(4e^{4x}) + e^{4x}(4C) + 4Ce^{4x}$

$= 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 6(4Cxe^{4x} - Ce^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$

$e^{4x}(16Cx - 24Cx + 8Cx + 4C + 4C - 6C) = 8e^{4x}$

$e^{4x}(0 + 2C) = 8e^{4x}$

$2C = 8e^{4x}$

$C = 4$

$y = 4xe^{4x}$

G_s

$Ae^{4x} + Be^{2x} + 4xe^{4x}$