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Question

Parametric equation of curve are given in equation ① & ②

$x = \cos t + t\sin t$ — ①

$y = \sin t - \cos t$ — ②

i) Determine an expression for radius of curvature.

ii) Expression for the coordinate (h, k) of the centre of curvature.

Soln

$y = \sin t - t\cos t$

$x = \cos t + t\sin t$

$dy/dt = \cos t - (-t\sin t + \cos t)$

$= \cos t + t\sin t - \cos t$

$= \cos t - \cos t + t\sin t$

$\frac{dy}{dt} = t\sin t$

$dx/dt = -\sin t + (t\cos t + \sin t)$

$= -\sin t + t\cos t + \sin t$

$\frac{dx}{dt} = t\cos t$

$dy/dx = \frac{dy}{dt} \times \frac{dt}{dx} = t\sin t \times \frac{1}{t\cos t}$

$= \frac{t\sin t}{t\cos t} = \tan t.$

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$$\frac{d^2y}{dx^2} = \frac{d\tan t}{dt} \times \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \sec^2 t \times \frac{1}{t\cos t} = \frac{1}{t} \times \frac{1}{\cos^3 t} = t^{-1}\sec^3 t$$

$$R = \frac{\left(1+\left(dy/dx\right)^2\right)^{3/2}}{d^2y/dx^2}$$

$$= \frac{\left(1+(\tan t)^2\right)^{3/2}}{t^{-1}\sec^3 t}$$

$$R_1 = \frac{(\sec t)^{2+3/2}}{t^{-1}\sec^3 t} = \frac{(\sec t)^3}{t^{-1}\sec^3 t}$$

$$R_1 = \frac{(\sec t)^3}{t^{-1}(\sec t)^3}$$

$$R_1 = \frac{\cancel{(\sec t)^3}}{t^{-1}\cancel{(\sec t)^3}} = \frac{1}{t^{-1}}$$

$R_1 = t$ units

Centre of curvature

$$x_c = h + R_1\sin\theta$$

$$h = x_1 - R_1\sin\theta$$

$$k = y + R_1\cos\theta$$

$$\theta = \tan^{-1}(dy/dx)$$

$$\theta = \tan^{-1}(\tan t)$$

$$= t$$

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$x_1 = \cos t + t \sin t$

$h = \cos t + t \sin t - (t) \sin t$.

$h = \cos t + t \sin t - t \sin t$

$h = \cos t$

$K = y_1 + R \cos \theta$

$Y_1 = \sin t - t \cos t$

$K = \sin t - t \cos t + t \cos t$

$K = \sin t - t \cos t + t \cos t$

$K = \sin t$

Centre of curvature.