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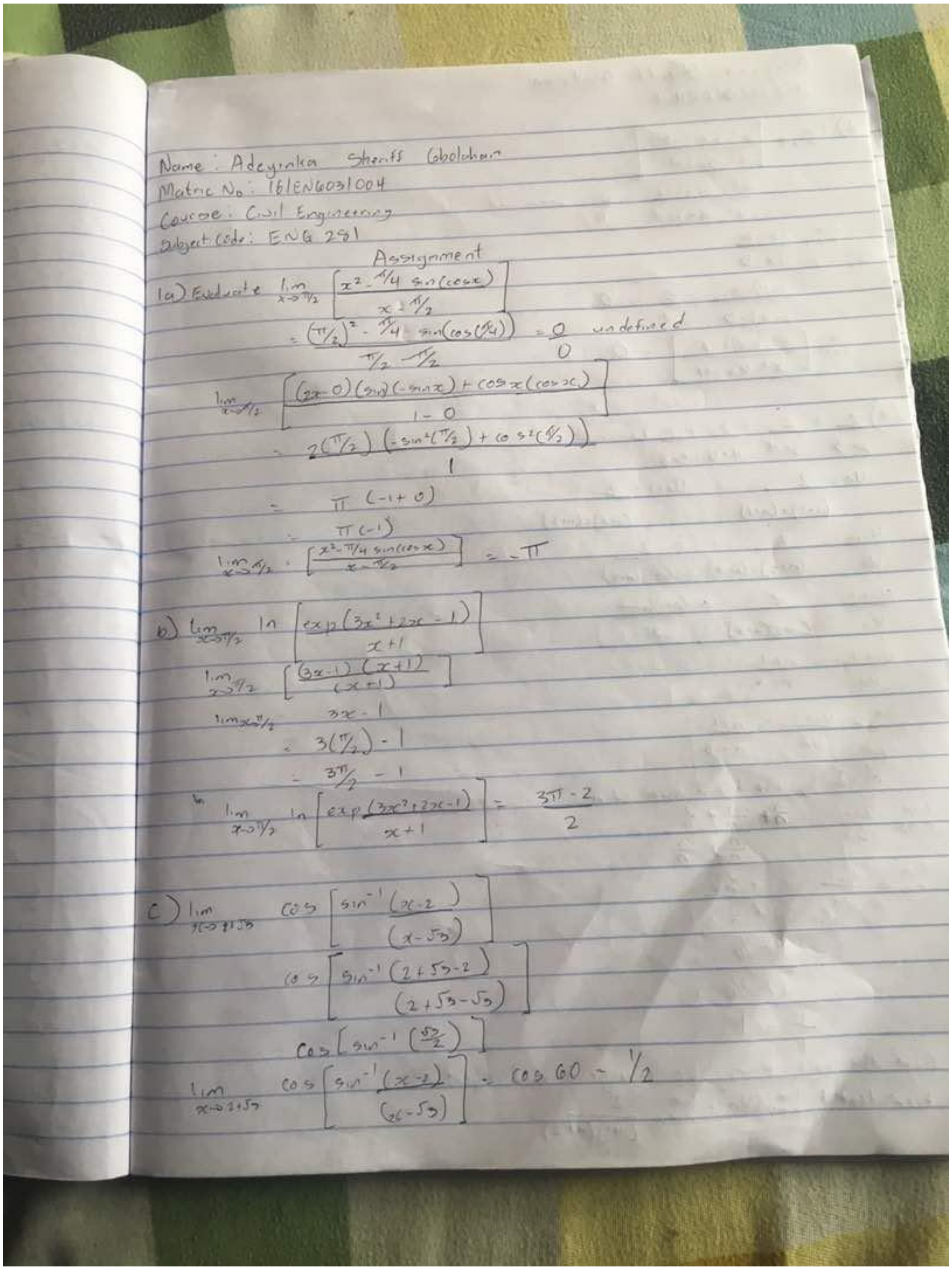
CIVIL ENGINEERING

16/ENG03/004

ENG281



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Matric No: 16/ENG03/004

Course: Civil Engineering

Subject Code: ENG 281

Assignment

1a) Evaluate $\lim_{x \to \pi/2} \left[\frac{x^2 - \pi/4 \sin(\cos x)}{x - \pi/2} \right]$

$$= \frac{(\pi/2)^2 - \pi/4 \sin(\cos(\pi/4))}{\pi/2 - \pi/2} = \frac{0}{0} \text{ undefined}$$

$$\lim_{x \to \pi/2} \left[\frac{(2x - 0)(\sin)(-\sin x) + \cos x(\cos x)}{1 - 0} \right]$$

$$= \frac{2(\pi/2)(-\sin^2(\pi/2) + \cos^2(\pi/2))}{1}$$

$$= \pi(-1 + 0)$$

$$= \pi(-1)$$

$$\lim_{x \to \pi/2} \left[\frac{x^2 - \pi/4 \sin(\cos x)}{x - \pi/2} \right] = -\pi$$

b) $\lim_{x \to \pi/2} \ln \left[\frac{\exp(3x^2 + 2x - 1)}{x + 1} \right]$

$$\lim_{x \to \pi/2} \left[\frac{(3x - 1)(x + 1)}{(x + 1)} \right]$$

$$\lim_{x \to \pi/2} 3x - 1$$

$$= 3(\pi/2) - 1$$

$$= 3\pi/2 - 1$$

$$\lim_{x \to \pi/2} \ln \left[\exp \frac{(3x^2 + 2x - 1)}{x + 1} \right] = \frac{3\pi - 2}{2}$$

c) $\lim_{x \to 2 + \sqrt{3}} \cos \left[\sin^{-1} \frac{(x - 2)}{(x - \sqrt{3})} \right]$

$$\cos \left[\sin^{-1} \frac{(2 + \sqrt{3} - 2)}{(2 + \sqrt{3} - \sqrt{3})} \right]$$

$$\cos \left[\sin^{-1} \left(\frac{\sqrt{3}}{2} \right) \right]$$

$$\lim_{x \to 2 + \sqrt{3}} \cos \left[\sin^{-1} \frac{(x - 2)}{(x - \sqrt{3})} \right] = \cos 60 = 1/2$$

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d) $\lim_{x \to 4} \left[\frac{x^2-8x+16}{x^2-5x+4}\right]$

$= \frac{4^2-8(4)+16}{4^2-5(4)+4} = \frac{0}{0}$ undetermined

$\lim_{x \to 4} \frac{2x-8}{2x-5}$

$= \frac{2(4)-8}{2(4)-5} = \frac{0}{3} = 0$

$\lim_{x \to 4} \left[\frac{x^2-8x+16}{x^2-5x+4}\right] = 0$

2a) $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \dots$

$U_n = \frac{2}{(n+1)\times(n+2)}$ $\quad U_{n+1} = \frac{2}{(n+2)\times(n+3)}$

$\frac{U_{n+1}}{U_n} = \frac{2}{(n+2)\times(n+3)} \div \frac{2}{(n+1)(n+2)}$

$\frac{U_{n+1}}{U_n} = \frac{\cancel{2}}{\cancel{(n+2)}(n+3)} \times \frac{(n+1)\cancel{(n+2)}}{\cancel{2}}$

$\frac{U_{n+1}}{U_n} = \frac{n+1}{n+3}$

$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{n+1}{n+3}$

divide by the highest power of n

$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{\frac{n}{n}+\frac{1}{n}}{\frac{n}{n}+\frac{3}{n}}$

$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{1+\frac{1}{n}}{1+\frac{3}{n}}$

$= \frac{1+1/0}{1+3/0}$

$= \frac{1}{1} = 1$

From the rule it may be convergent or divergent

From test 1 $\lim_{n \to \infty} U_n = \lim_{n \to \infty} \frac{2}{(n+1)(n+2)}$

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$$U_n = \lim_{n\to\infty} \frac{2}{(n^2+3n+2)}$$

$$U_n = \lim_{n\to\infty} \frac{2/n^2}{n^2/n^2 + 3n/n^2 + 2/n^2}$$

$$= \frac{2/0}{1 + 0/0 + 2/0}$$

$$= \frac{0}{1} = 0$$

The series is covergent

2b) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \dots$

$U_n = \frac{2}{n^2}$ $\quad U_{n+1} = \frac{2}{(n+1)^2}$ $\quad U_{n+1} = \frac{2}{n^2+2n+1}$

$$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{n^2+2n+1}{n^2}$$

$$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{n^2/n^2 + 2n/n^2 + 1/n^2}{n^2/n^2}$$

$$= \frac{1 + \frac{(2\times 0)}{0} + 1/0}{1}$$

$$= \frac{1+0+0}{1}$$

$$= 1$$

From the rule it may be covergent or divergent

From test 1 $\lim_{n\to\infty} U_n = \lim_{n\to\infty} \frac{2}{n^2}$

$$= \lim_{n\to\infty} \frac{4n^2}{n^2/n^2} = \frac{2/0}{1} = \frac{0}{1}$$

$$= 0$$

The series is covergent

2c) $U_n = \frac{1+2n^2}{1+n^2}$

$$\lim_{n\to\infty} U_n = 0$$

$$\lim_{n\to\infty} \frac{1+2n^2}{1+n^2} = \frac{1/n^2 + 2n^2/n^2}{1/n^2 + n^2/n^2}$$

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as $n \to \infty$ $1/n^2 = 0$

$$= \frac{0+2}{0+1} = 2$$

The series is divergent

3) $\frac{x}{27} + \frac{x^2}{125} + \dots \frac{x^n}{(2n+1)^3}$

$$U_n = \frac{x^n}{(2n+1)^3}, \quad U_{n+1} = \frac{x^{n+1}}{(2n+2)^3}$$

$$\lim \frac{U_{n+1}}{U_n} = \frac{x^{n+1}}{(2n+2)^3} \times \frac{(2n+1)^3}{x^n}$$

$$= \frac{x(2n+1)^3}{(2n+2)^3} \Rightarrow \frac{x(8n^3+12n^2+n+1)}{8n^3+24n^2+24n+8}$$

$$= \frac{x\left(\frac{8n^3}{n^3} + \frac{12n^2}{n^3} + \frac{n}{n^3} + \frac{1}{n^3}\right)}{\left(\frac{8n^3}{n^3} + \frac{24n^2}{n^3} + \frac{24n}{n^3} + \frac{8}{n^3}\right)}$$

$$= \frac{x(8 + 12/n + 1/n^2 + 1/n^3)}{(8 + 24/n + 24/n^2 + 8/n^3)}$$

as $n \to \infty$, $1/n \to 0$

$$\frac{x(8+0+0+0)}{(8+0+0+0)} = \frac{8x}{8} \Rightarrow x = 1$$

$\therefore x \leq 1$

4) $\lim\limits_{x \to 0} \left[\frac{\sin x - \cos x}{x^3}\right]$

By using L'hospital's rule

$$y = \left[\frac{\sin x - \cos x}{x^3}\right]$$

$$\frac{dy}{dx} = \left[\frac{\cos x + \sin x}{3x^2}\right]$$

$$\frac{d^2y}{dx^2} = \frac{-\sin x + \cos x}{6x}$$

$$\frac{d^3y}{dx^3} = \frac{-\cos x - \sin x}{6}$$

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$$\lim_{x \to 0} = \frac{-\cos 0 - \sin 0}{6} = \frac{-1-0}{6} = {}^{1}/_{6}$$

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right] = -{}^{1}/_{6}$$