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1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$m^2 - m - 2 = 0$

$m^2 + m - 2m - 2 = 0$

$m(m+1) - 2(m+1) = 0$

$(m-2)(m+1) = 0$

$m_1 = 2, \quad m_2 = -1$

$y = Ae^{2x} + Be^{-x}$

For P.I : $y = c$

$dy/dx = 0$

$d^2y/dx^2 = 0$

$(0) - (0) - 2y = 8$

$-2y = 8$

$y = -4$

∴ General solution $= y = Ae^{2x} + Be^{-x} - 4$

2) $\frac{d^2y}{dx^2} + 4y = 10e^{3x}$

CF : $\frac{d^2y}{dx^2} - 4y = 0$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm\sqrt{4} = \pm 2$

$m = \alpha + j\beta$

~~$y = e^{\alpha x}(A\cos 2x + B\sin 2x)e^{2x}$~~

$y = e^{\alpha x}(A\cos \beta x + B\sin \beta x)$

$y = e^{2x}(A\cos 2x + B\sin 2x)$

For P.I : $y = Ce^{3x}$

$dy/dx = 3Ce^{3x}$

$d^2y/dx^2 = 9Ce^{3x}$

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$5Ce^{3x} = 10Ce^{3x}$

$C = \frac{10e^{3x}}{5e^{3x}}$

$C = 2 \quad y = 2e^{3x}$

$\therefore G.S = y = e^{x}(A\cos 2x + B\sin 2x) + 2e^{3x}$

(3) $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = e^{-2x}$

CF: $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = 0$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1) = 0$

$m = -1$ twice

$y = e^{-x}(A + Bx)$

PI: $y = Ce^{-2x}$

$dy/dx = -2Ce^{-2x}$

$d^2y/dx^2 = 4Ce^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

$C = 1$

$y = e^{-2x}$

GS: $y = e^{-x}(A + Bx) + e^{-2x}$

(4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

CF: $y = \frac{d^2y}{dx^2} + 25y = 0$

$m^2 + 25 = 0$

$m^2 = -25$

$m = \pm\sqrt{-25}$

$m = \pm j5$

$m = \alpha \pm j\beta$

$y = e^{x}(A\cos 5x + B\sin 5x)$

PI: $y = 5x^2 + x$

$y = Cx^2 + Dx + E$

$dy/dx = 2Cx + D$

$d^2y/dx^2 = 2C$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

By comparing coefficients.

$25C = 5$

$C = 1/5$

$25D = 1$

$D = 1/25$

$25E + 25E = 0$

$25E + 2/5 = 0$

$25E = -2/5$

$E = -2/125$

$y = 1/5\,x^2 + 1/25\,x - 2/125$

$\therefore$ GS: $y = e^{-x}(A\cos 5x + B\sin 5x) + 1/5\,x^2 + 1/25\,x - 2/125$

(5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

CF: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$(m-1)(m-1) = 0$

$m = 1$ twice

$y = e^{x}(A + Bx)$

PI: $y = C\cos x + D\sin x$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$d^2y/dx^2 = -C\cos x - D\sin x$

$-C\cos x - D\sin x - 2(-C\cos x - D\sin x) + (C\cos x + D\sin x) = 4\sin x$

$-C\cos x - 2D\cos x + C\cos x - D\sin x + 2C\sin x + D\cos x = 4\sin x$

$-2D\cos x + 2C\sin x = 4\sin x$

$2C = 4$

$C = 2$

$D = 0$

$y = 2\cos x$

GS: $y = e^{ax}(A + Bx) + 2\cos x$

b) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$ at $x=0$, $y=1$, $dy/dx = -2$

CF: $M^2 + 4M + 5 = 0$

$a=1, b=4, c=5$

$M = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$

$= \frac{-4 \pm \sqrt{16 - 20}}{2}$

$= \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2j}{2}$

$M = -2 \pm j$

$y = e^{-2x}(C\cos x + D\sin x)$

PI: $y = Cxe^{-2x}$

$dy/dx = -2Cxe^{-2x} + Ce^{-2x}$

$d^2y/dx^2 = 4Cxe^{-2x} - 2Ce^{-2x} - 2Ce^{-2x}$

$4Cxe^{-2x} + 4(-2Cxe^{-2x} + Ce^{-2x}) + 5Cxe^{-2x} = 2e^{-2x}$

$4Cxe^{-2x} - 8Cxe^{-2x} + 4Ce^{-2x} + 5Cxe^{-2x} = 2e^{-2x}$

$Cxe^{-2x} + 4Ce^{-2x} = 2e^{-2x}$

$C = 2$

$y = 2xe^{-2x}$

GS: $y = e^{-2x}(C\cos x + D\sin x) + e^{-2x}$

At $y=1$, $x=0$

$1 = e^{-2(0)}(C\cos(0) + D\sin(0)) + e^{-2(0)}$

$1 = 1(C + 0) + 1$

$1 = C + 1$, $C = 0$

$$\frac{dy}{dx} = -2e^{-2x}(-C\sin x + D\sin x) - 2e^{-2x}$$

$$-2 = -2e^{-2(0)}(-C\sin(0) + D\sin(0)) - 2e^{-2(0)}$$

$$-2 = -2(0 + D) - 2$$

$$-2 = -2D - 2$$

$$2D = -2 + 2$$

$$D = 0$$

(7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

CF: $3m^2 - 2m - 1 = 0$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{(-2)^2 - 4(3)(-1)}}{2(3)}$$

$$= \frac{2 \pm \sqrt{4 + 12}}{6}$$

$$= \frac{2 \pm \sqrt{16}}{6} = \frac{2 \pm 4}{6}$$

$$m_1 = \frac{2+4}{6} \quad \text{or} \quad m_2 = \frac{2-4}{6}$$

$$m_1 = 1 \quad \text{or} \quad m_2 = -1/3$$

$$y = Ae^x + Be^{-1/3x}$$

PI: $y = Cx + D$

$dy/dx = C$

$d^2y/dx^2 = 0$

$$3(0) - 2(C) - Cx - D = 2x - 3$$

$$-2C - Cx - D = 2x - 3$$

$$-C = 2$$

$$C = -2$$

$$-2C - D = -3$$

$$-2(-2) - D = -3$$

$$4 - D = -3$$

$$D = 7 \qquad y = -2x + 7$$

GS: $y = Ae^x + Be^{-1/3x} - 2x + 7$

5) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

CF : $m^2 - 6m + 8 = 0$

$m^2 - 4m - 2m + 8 = 0$

$m(m-4) - 2(m-4) = 0$

$(m-2)(m-4) = 0$

$m = 2$ or $m = 4$

$m_1 = 2, m_2 = 4$

$y = Ae^{2x} + Be^{4x}$

PI : $y = 8e^{4x}$

$y = Cxe^{4x}$

$dy/dx = 4Cxe^{4x} + Ce^{4x}$

$d^2y/dx = 16Cxe^{4x} + 4Ce^{4x}$

$4Ce^{4x} + 16Cxe^{4x} + 4Ce^{4x} - 6(Ce^{4x} + 4Cxe^{4x}) + 8(Cxe^{4x})$

$= 8e^{4x}$

$4Ce^{4x} + 4Ce^{4x} - 6Ce^{4x} + 8Cxe^{4x} + 16Cxe^{4x} - 24Cxe^{4x}$

$4Ce^{4x} + 4Ce^{4x} - 6Ce^{4x} = 8e^{4x}$

$2Ce^{4x} = 8e^{4x}$

$2C = 8$

$C = 4$

$y = 4xe^{4x}$

$\therefore$ GS : $y = Ae^{2x} + Be^{4x} + 4xe^{4x}$