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Assignment 1

1. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$y'' - y' - 2y = 8$

$k^2 - k - 2 = 0$

$(k^2 - 2k)(+k - 2) = 0$

$k(k-2) + 1(k-2) = 0$

$(k-2)(k+1) = 0$

$k_2 = -1, k_1 = 2$

$y = C_1 e^{2x} + C_2 e^{-x}$

$y_p = A$

$y'_p = 0$

$y_p'' = 0$

$0 - 0 - 2A = 8$

$-2A = 8$

$A = \frac{-8}{2}$ $A = -4$ $\therefore y_p = -4$

$y_c = y_h + y_p$

$y = C_1 e^{2x} + C_2 e^{-x} - 4$

Question 2

2. $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$y'' - 4y = 10e^{3x}$

$k^2 - 4 = 0$

$k^2 = 4$

$k = \pm\sqrt{4}$

$k = 2$ or $-2$

$k_1 = 2$ and $k_2 = -2$

$y_h = C_1 e^{2x} + C_2 e^{-2x}$

$y_p = Ae^{3x}$

$y_p' = 3Ae^{3x}$

$y_p'' = 9Ae^{3x}$

$9Ae^{3x} - 4(Ae^{3x}) = 10e^{3x}$

$9Ae^{3x} - 4Ae^{3x} = 10e^{3x}$

$9A - 4A = 10$

$+5A = 10$

$A = \frac{+10}{5}$

$y_p = \frac{+10}{5} e^{3x} = 2e^{3x}$

$y = y_h + y_p$

$y = c_1 e^{2x} + c_2 e^{-2x} + 2e^{3x}$

Question 3

5 $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

$y'' + 2y' + y = e^{-2x}$

$k^2 + 2k + 1 = 0$

$(k^2 + k)(k + 1) = 0$

$k(k+1) + 1(k+1) = 0$

$(k+1)(k+1) = 0$

$k_1 = -1$ and $k_2 = -1$

$y_h = c_1 y_1 + x c_2 y_2$

$y_h = c_1 e^{-x} + x c_2 e^{-x}$

$y_p = Ae^{-2x}$

$y_p' = -2Ae^{-2x}$

$y_p'' = 4Ae^{-2x}$

$4Ae^{-2x} + 2(-2Ae^{-2x}) + Ae^{-2x} = e^{-2x}$

$4Ae^{-2x} - 4Ae^{-2x} + Ae^{-2x} = e^{-2x}$

$Ae^{-2x} = e^{-2x}$

$A = 1$

$\therefore y_p = e^{-2x}$

$y = y_h + y_p$

$y = C_1 e^{-2x} + (C_2 e^{-x} + e^{-2x}$

$y = e^{-x}[C_1 + C_2 x] + e^{-2x}$ //

Question 4

A $\frac{d^2y}{dx^2} + 25y = 5x^2 + 2x$

$y'' + 25y = 5x^2 + 2x$

$k^2 + 25 = 0$

$k^2 = -25$

$k = \pm\sqrt{-25}$

$k_1 = 5i$ and $k_2 = -5i$

$y_h = C_1 e^{5xi} + C_2 e^{-5xi}$

$y_h = A\cos 5x + B\sin 5x$

$y_p = Ax^2 + Bx + C$

$y'_p = 2Ax + B$

$y''_p = 2A$

$2A + 25Ax^2 + 25Bx + 25C = 5x^2 + 2x$

$(25A)x^2 + (25B)x + (2A + 25C) = (5)x^2 + (1)x + 0$

$25A = 5$

$A = 5/25 = 1/5$

$25B = 1$

$B = \frac{1}{25}$

$2A + 25C = 0$

$25C = -2A$, $25C = -2\left(\frac{1}{5}\right) = \frac{-2}{5}$

$C = \frac{-2}{5} \times \frac{1}{25}$ $= \frac{-2}{125}$

$y_h = \frac{1}{5}x^2 + \frac{1}{25}x - 2/125$

$y = y_h + y_p$

$y = A\cos 5x + B\sin 5x + \frac{1}{5}x^2 + 1/25x - 2/125$

Question 5

5 $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$y'' - 2y' + y = 4\sin x$

$k^2 - 2k + 1 = 0$

$(k^2 - k)(-1k + 1) = 0$

$1k(k-1) - 1(k-1) = 0$

$(k-1)(k-1) = 0$

$k_1 = 1$ and $k_2 = 1$

$y_h = C_1 y_1 + x C_2 y_2$

$y_h = C_1 e^x + x C_2 e^x$

$y_p = A\cos x + B\sin x$

$y'_p = -A\sin x + B\cos x$

$y''_p = -A\cos x - B\sin x$

$-A\cos x - B\sin x - 2[-A\sin x + B\cos x] + A\cos x + B\sin x = 4\sin x$

$-A\cos x - B\sin x + 2A\sin x - 2B\cos x + A\cos x + B\sin x = 4\sin x$

$(2A)\sin x + (-2B)\cos x = (4)\sin x + (0)\cos x$

$2A = 4$

$A = 4/2 = 2$

$-2B = 0$

$B = 0$

$y_p = 2\cos x + 0\sin x = 2\cos x$

$y = y_h + y_p$

$y = C_1 e^x + x C_2 e^x + 2\cos x$

$y = e^x[C_1 + C_2 x] + 2\cos x$

$$y_h = C_1 e^{(-2+i)x} + C_2 e^{(-2-i)x}$$

$$y_h = C_1 e^{-2x+ix} + C_2 e^{-2x-ix}$$

$$y_h = C_1 e^{-2x+ix} + C_2 e^{-2x} \cdot e^{-ix}$$

$$y_h = e^{-2x}\left[C_1 e^{ix} + C_2 e^{-ix}\right]$$

$$y_h = e^{-2x}\left[A\cos x + B\sin x\right]$$

$$y_p = Ae^{-2x}$$

$$y'_p = -2Ae^{-2x}$$

$$y''_p = 4Ae^{-2x}$$

$$4Ae^{-2x} + 4\left(-2Ae^{-2x}\right) + 5\left(Ae^{-2x}\right) = 2e^{-2x}$$

$$4Ae^{-2x} - 8Ae^{-2x} + 5Ae^{-2x} = 2e^{-2x}$$

$$Ae^{-2x} = 2e^{-2x}$$

$$A = 2$$

$$y_p = 2e^{-2x}$$

$-2 = -2A + B - 4$

$B = -2 + 2A + 4$

$B = -2 + 2(-1) + 4$

$B = -2 - 2 + 4$

$B = 0$

$y = e^{-2x}[-\cos x + 0 \sin x] + 2e^{-2x}$

$y = -e^{-2x}\cos x + 2e^{-2x}$

$y = e^{-2x}[2 - \cos x]$

## Question 7

a) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3y'' - 2y' - y = 2x - 3$

$3k^2 - 2k - 1 = 0$

$(3k^2 - 3k)(k - 1) = 0$

$3k(k-1) + 1(k-1) = 0$

$(k-1)(3k+1) = 0$

$k_1 = 1$ and $k_2 = -\frac{1}{3}$

$y_h = C_1 e^{x} + C_2 e^{-x/3}$

$y_p = Ax + B$

$y'_p = A$

$y''_p = 0$

$3(0) - 2(A) - (Ax + B) = 2x - 3$

$-2A - Ax - B = 2x - 3$

$(-A)x + (-2A - B) = (2)x + (-3)$

$-A = 2$

$A = -2$

$-2A - B = -3$

$-2(-2) - B = -3$

$B = 4 + 3 = 7$

$y_p = -2x + 7$

$y = y_h + y_p, \quad y = C_1 e^{x} + C_2 e^{-x/3} - 2x + 7$

Question 8

8 $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$y'' - 6y' + 8y = 8e^{4x}$

$k^2 - 6k + 8 = 0$

$(k^2 - 4k)(-2k + 8) = 0$

$k(k-4) - 2(k-4) = 0$

$(k-4)(k-2) = 0$

$k_1 = 4, \quad k_2 = 2$

$y_h = c_1 e^{4x} + c_2 e^{2x}$

$y_p = Axe^{4x}$

$y'_p = A[e^{4x} + 4xe^{4x}] = Ae^{4x} + 4Axe^{4x}$

$y''_p = 4Ae^{4x} + 4A[e^{4x} + 4xe^{4x}]$

$y''_p = 4Ae^{4x} + 4Ae^{4x} + 16xe^{4x} = 8Ae^{4x} + 16Axe^{4x}$

$8Ae^{4x} + 16Axe^{4x} - 6(Ae^{4x} + 4Axe^{4x}) + 8(Axe^{4x}) = 8e^{4x}$

$8Ae^{4x} + 16Axe^{4x} - 6Ae^{4x} - 24Axe^{4x} + 8Axe^{4x} = 8e^{4x}$

$2Ae^{4x} = 8e^{4x}$

$2A = 8$

$A = 4$

$\therefore y_p = 4xe^{4x}$

$y = y_h + y_p$

$y = c_1 e^{4x} + c_2 e^{2x} + 4xe^{4x}$