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1a $\lim_{x \to \frac{\pi}{2}} \left[\frac{(x^2 - \frac{\pi}{4}) \sin(\cos x)}{x - \frac{\pi}{2}} \right]$

Solution.

Using product rule for the numerator,

Let $u = x^2 - \frac{\pi}{4}$ and $r = \sin(\cos x)$

$\therefore \frac{dy}{dx} = u \cdot \frac{dr}{dx} + r \cdot \frac{du}{dx}$

For $u = x^2 - \frac{\pi}{4}$

$\frac{du}{dx} = 2x$

For $r = \sin(\cos x)$

$w = \cos x \quad r = \sin w$

$\frac{dw}{dx} = -\sin x \quad \frac{dr}{dw} = \cos w$

$\frac{dr}{dx} = \frac{dw}{dx} \times \frac{dr}{dw} = -\sin x \times \cos x$

$\frac{dr}{dx} = -\sin(\cos(\cos x))$

$\frac{dy}{dx} = \left[x^2 - \frac{\pi}{4}\right] \cdot -\cos(\cos x)\sin x + \sin(\cos x) \cdot 2x$

At denominator $y = x - \pi/2$

$\frac{dy}{dx} = 1$

$\therefore \frac{x^2 - \frac{\pi}{4}(-\cos(\cos x)\sin x) + \sin(\cos x)2x}{1}$

$\lim x \to \frac{\pi}{2} \left(\frac{\pi^2}{2^2} - \frac{\pi}{4}\right)\left(-\cos(\cos\frac{\pi}{2})\sin\frac{\pi}{2}\right) + \sin(\cos\frac{\pi}{2}) \cdot \pi$

$\left(\frac{\pi^2}{4} - \frac{\pi}{4}\right)(-1) + 0$

$= \left(\frac{\pi^2 - \pi}{4}\right) \cdot -1 = \frac{-\pi^2 + \pi}{4}$

$= \frac{\pi - (-\pi + 1)}{4}$

(b) $\lim_{x \to \frac{\pi}{2}} \ln\left[\frac{\exp(3x^2+2x-1)}{x+1}\right]$

Solution

$$\frac{3\left(\frac{\pi}{2}\right)^2 + 2\left(\frac{\pi}{2}\right) - 1}{\frac{\pi}{2}+1}$$

$$= \frac{\frac{3\pi^2}{4} + \pi - 1}{\frac{\pi+2}{2}} = \frac{3\pi^2+4\pi-4}{4} \times \frac{2}{\pi+2}$$

$$= \frac{3\pi^2+4\pi-4}{2(\pi+2)}$$

$$\frac{3\pi^2+4\pi-4}{2(\pi+2)} = \left(\frac{\pi}{1} - \frac{3}{3}\right) \times \frac{1}{2}$$

$$= \frac{3\pi-2}{3} \times \frac{1}{2} = \frac{3\pi-2}{6}$$

(c) $\lim_{x \to 2+\sqrt{3}} \cos\left[\frac{\sin^{-1}(x-2)}{(x-\sqrt{3})}\right]$

Solution

$$\cos\left[\frac{\sin^{-1}((2+\sqrt{3})-2)}{2+\sqrt{3}-\sqrt{3}}\right]$$

$$= \cos\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right] \quad \text{Rem. } \cos(60) = 1/2$$

$$\lim_{x \to 2+\sqrt{3}} = \cos\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right] = \frac{1}{2}$$

(d) $\lim_{x \to 4} \left(\frac{x^2-8x+16}{x^2-5x+4}\right)$

Solution

$$\lim_{x \to 4} = \cancel{\frac{4^2+8(4)+16}{4^2-5(4)+4}} \quad \left[\frac{2x-8}{2x-5}\right]$$

$$= \cancel{16-32+16} = \left[\frac{2(4)-8}{2(4)-5}\right] = \frac{0}{3} = 0$$

$$\therefore \lim_{x \to 4}\left(\frac{x^2-8x+16}{x^2-5x+4}\right) = 0$$

2(a) $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \cdots$

Using standard series

$\frac{2}{2^p} + \frac{2}{3^p} + \frac{2}{4^p} + \frac{2}{5^p} + \cdots$

when $P = 2$, we get

$\frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \frac{2}{5^2} + \cdots$

Since

$\frac{2}{2\times3} < \frac{2}{2^2}$; $\frac{2}{3\times4} < \frac{2}{3^2}$; $\frac{2}{4\times5} < \frac{2}{4^2}$

Therefore the series is convergent.

(b) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$

Solution.

Use comparison test,

when $P = 2$

$\frac{2}{1^p} + \frac{2}{2^p} + \frac{2}{3^p} + \frac{2}{4^p}$

$P > 1$, makes the series convergent.

(c) $U_n = \frac{1+2n^2}{1+n^2}$

Solution

$U_n = \frac{1+2n^2}{1+n^2}$

$U_n = \lim \left[\frac{1/n^2 + 2n^2/n^2}{1/n^2 + n^2/n^2}\right]$

$= \left[\frac{1/n^2 + \frac{2}{1}}{1/n^2 + 1}\right]$

$= \frac{0+2}{0+1}$

$= \underline{\underline{2}}$

$\therefore \lim U_n = 2$

and $n \Rightarrow$ infinity.

Since $\lim U_n \neq 0$
Therefore the series is divergent.

(3) Find the range of values of x for which the series below is absolutely convergent.

Solution

$$\frac{x}{27} + \frac{x^2}{125} + \dots + \frac{x^n}{(2n+1)^3}$$

$$|U_n| = \frac{x^n}{(2n+1)^3}$$

$$|U_{n+1}| = \frac{x^{n+1}}{(2n+3)^3}$$

$$\frac{|U_{n+1}|}{|U_n|} = \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$$

$$\frac{x^n \times x}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$$

$$= \frac{x(2n+1)^3}{(2n+3)^3}$$

$$= \frac{x(2n+1)(2n+1)(2n+1)}{(2n+3)(2n+3)(2n+3)}$$

$$= \frac{x(8n^3+12n^2+6n+1)}{8n^3+36n^2+54n+27}$$

$$= \frac{x\left[\frac{8n^3}{n^3} + \frac{12n^2}{n^3} + \frac{6n}{n^2} + \frac{1}{n^3}\right]}{\frac{8n^3}{n^3} + \frac{36n^2}{n^3} + \frac{54n}{n^2} + \frac{27}{n^3}}$$

$$= \frac{x\left[8 + 12/n + 6/n^2 + 1/n^3\right]}{\left[8 + 36/n + 54/n^2 + 27/n^3\right]}$$

$$= \frac{x[8+0+0+0]}{8+0+0+0}$$

$$= \frac{x8}{8} = x$$

$\therefore \lim_{n \to \alpha} \left| \frac{U_{n+1}}{U_n} \right| = x$

For absolute convergence $\lim_{n \to \alpha} \left| \frac{U_{n+1}}{U_n} \right| < 1$

A series is said to be convergent when $-1 \leq x \leq 1$

(4) Using L'Hopital's Rule Evaluate

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$$

Solution

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$$

$$= \lim_{x \to 0} \left[\frac{\cos x + \sin x}{3x^2} \right]$$

$$\lim_{x \to 0} \left[\frac{\cos x + \sin x}{3x^2} \right]$$

$$= \left[\frac{-\sin x + \cos x}{6x} \right]$$

$$= \lim_{x \to 0} \left[\frac{-\cos x + (-\sin x)}{6} \right]$$