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Mechanical Engineering
16/ENG06/020
ENG 281

1a Evaluate $\lim_{x \to \pi/2} \left[\frac{x^2 - \pi/4 \sin(\cos x)}{x - \pi/2} \right]$

$$= \frac{(\pi/2)^2 - \pi/4 \sin(\cos(\pi/4))}{\pi/2 - \pi/2} = \frac{0}{0} \text{ Undefined}$$

$$\lim_{x \to \pi/2} \left[\frac{(2x-0)(\sin)(-\sin x) + \cos x(\cos x)}{1-0} \right]$$

$$= \frac{2(\pi/2)\left[-\sin^2(\pi/2) + \cos^2(\pi/2)\right]}{1}$$

$$= \pi(-1+0)$$

$$= \pi(-1)$$

$$\lim_{x \to \pi/2} \left[\frac{x^2 - \pi/4 \sin\cos(x)}{x - \pi/2} \right] = -\pi$$

b) $\lim_{x \to \pi/2} \ln \left[\frac{\exp(3x^2 + 2x - 1)}{x+1} \right]$

$$\lim_{x \to \pi/2} \left[\frac{(3x-1)(x+1)}{(x+1)} \right]$$

$$\lim_{x \to \pi/2} (3x-1)$$

$$= 3(\pi/2) - 1$$

$$= 3\pi/2 - 1$$

$$\lim_{x \to \pi/2} \ln \left[\frac{\exp(3x^2 + 2x - 1)}{x+1} \right] = \frac{3\pi - 2}{2}$$

c $\lim_{x \to 2+\sqrt{3}} \cos \left[\sin^{-1} \frac{(x-2)}{(x-\sqrt{3})} \right]$

$$\cos \left[\sin^{-1} \frac{(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})} \right]$$

$$\cos \left[\sin^{-1} \left(\frac{\sqrt{3}}{2} \right) \right]$$

$$\lim_{x \to 2+\sqrt{3}} \cos \left[\sin^{-1} \frac{(x-2)}{(x-\sqrt{3})} \right] = \cos 60 = \frac{1}{2}$$

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d) $\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right]$

$= \frac{4^2 - 8(4) + 16}{4^2 - 5(4) + 4} = \frac{0}{0}$ undefined

$\lim_{x \to 4} \frac{2x - 8}{2x - 5}$

$= \frac{2(4) - 8}{2(4) - 5} = \frac{0}{3} = 0$

$\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right] = 0$

2a) $\frac{2}{2 \times 3} + \frac{2}{3 \times 4} + \frac{2}{4 \times 5} + \frac{2}{5 \times 6} + \dots$

$U_n = \frac{2}{(n+1) \times (n+2)}$, $U_{n+1} = \frac{2}{(n+1+1)(n+2+1)} = \frac{2}{(n+2)(n+3)}$

$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{2}{(n+2)(n+3)} \times \frac{(n+1)(n+2)}{2}$

$= \frac{n^2 + 3n + 2}{n^2 + 5n + 6}$

divide through by the highest power of n

$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{n^2/n^2 + 3n/n^2 + 2/n^2}{n^2/n^2 + 5n/n^2 + 6/n^2}$

$= \frac{1 + 0 + 0}{1 + 0 + 0} = \frac{1}{1} = 1$

It maybe either convergent or divergent using test 1 on U_n

$U_n = \frac{2}{n^2 + 3n + 2}$

$= \frac{2/n^2}{n^2/n^2 + 3n/n^2 + 2/n^2} = \frac{0}{1 + 0 + 0}$

$= 0$ converge

b) $\frac{2}{1^2} + \frac{2^2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$

$U_n = \frac{2}{n^2}$, $U_{n+1} = \frac{2}{(n+1)^2}$

$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{2}{(n+1)^2} \times \frac{n^2}{2}$

$= \frac{n^2}{n^2+2n+1}$

divide by the highest power of n

$= \frac{n^2/n^2}{n^2/n^2 + 2n/n^2 + 1/n^2} = \frac{1}{1+0+0} = \frac{1}{1}$. mey convay or diverge

Using test 1 on U_n

$U_n = \frac{2}{n^2}$

$\frac{2/n^2}{n^2/n^2} = \frac{0}{1} = 0$ converge

c) $U_n = \frac{1+2n^2}{1+n^2}$

$\lim_{n \to \infty} U_n = \frac{1/n^2 + 2n^2/n^2}{1/n^2 + n^2/n^2}$

$= \frac{0+2}{0+1} = \frac{2}{1} = 2$ diverge

3) $\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3}$

$U_n = \frac{x^n}{(2n+1)^3}$, $U_{n+1} = \frac{x^{n+1}}{(2(n+1)+1)^3} = \frac{x^{n+1}}{(2n+3)^3}$

$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{2^n}$

$= \frac{(x)(2n+1)^3}{(2n+3)^3}$

$$= \frac{(8n^3 + 12n^2 + 6n + 1)x}{8n^3 + 18n^2 + 54n + 27}$$

$$= \frac{8xn^3 + 12xn^2 + 6xn + x}{8n^3 + 18n^2 + 54n + 27}$$

divide by highest power of n.

$$= \frac{8xn^3/n^3 + 12xn^2/n^3 + 6xn/n^3 + x/n^3}{8n^3/n^3 + 18n^2/n^3 + 54n/n^3 + 27/n^3}$$

$$= \frac{8x}{8}$$

$$\frac{8x}{8} < 1$$

$$8x < 8$$

$$x < 1$$

4) $$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$$

$$\frac{\sin 0 - \cos 0}{0^3} = \frac{0-1}{0} \quad \text{Undefined}$$

$$\lim_{x \to 0} \frac{\cos 0 + \sin 0}{3(0)^2} = \frac{1+0}{0} \quad \text{Undefined}$$

$$\lim_{x \to 0} \frac{-\sin x + \cos x}{6x} = \frac{-\sin 0 + \cos 0}{6(0)} = \frac{-0+1}{0} \quad \text{Undefined}$$

$$\lim_{x \to 0} \frac{-\cos x - \sin x}{6} = \frac{-\cos 0 - \sin 0}{6} = \frac{-1-0}{6} = -\frac{1}{6}$$