Haruna Muhammad
Electrical / Electronics
16/ENG04/023 16/ENG04/023
ENG 281

1a) Evaluate $\lim_{x \to \pi/2} \left[\frac{x^2 - \pi/4 \sin(\cos x)}{x - \pi/2} \right]$

$$\frac{(\pi/2)^2 - \pi/4 \sin(\cos(\pi/4)}{\pi/2 - \pi/2} = 0/0$$

$$\lim_{x \to \pi/2} \left[\frac{2x - 0)\ (\sin)(-\sin x) + \cos x\ (\cos x)}{1 - 0} \right]$$

$$\frac{2(\pi/2)\ [-\sin^2(\pi/2) + \cos^2(\pi/2)}{1}$$

$$\pi\ (-1 + 0)$$

$$\pi\ (-1)$$

$$\lim_{x \to \pi/2} \left[\frac{x^3 - \pi/4 \sin\cos(x)}{x - \pi/2} \right] = -\pi$$

$$\pi(-1)$$

$$\lim_{x\to \pi/2}\left[\frac{x^3-\pi/4\,\sin\cos(x)}{x+-\pi/2}\right]=-\pi$$

b) $$\lim_{x\to \pi/2} \ln\left[\frac{\exp(3x^2+2x-1)}{x+1}\right]$$

$$\lim_{x\to \pi/2}\left[\frac{(3x-1)(x+1)}{(x+1)}\right]$$

$$\lim_{x\to \pi/2}(3x-1)$$

$$3\left(\pi/2\right)-1$$

$$3\pi/2-1$$

$$\lim_{x\to \pi/2} \ln\left[\frac{\exp(3x^2+2x-1)}{x+1}\right]=\frac{3\pi-2}{2}$$

e)

c) $\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1} \frac{(x-2)}{(x-\sqrt{3})}\right]$

$\cos\left[\sin^{-1} \frac{(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})}\right]$

$\cos\left[\sin^{-1}\left(\sqrt{3}/2\right)\right]$

$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1} \frac{(x-2)}{(x-\sqrt{3})}\right] = \cos 60 = 1/2$

d) $\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4}\right]$

$\frac{4^2 - 8(4) + 16}{4^2 - 5(4) + 4} = \frac{0}{0}$ {undefined}

$\lim_{x \to 4} \frac{2x-8}{2x-5}$

$\frac{2(4) - 8}{2(4) - 5} = 0/3 = 0$

$\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4}\right] = 0$

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$$\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right] = 0$$

2a) $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \cdots$

$U_n = \frac{2}{(n+1)\times(n+2)}$, $U_{n+1} = \frac{2}{(n+1+1)(n+2+1)} = \frac{2}{(n+2)(n+3)}$

$$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{2}{(n+2)(n+3)} \times \frac{(n+1)(n+2)}{2}$$

$$\frac{n^2 + 3n + 2}{n^2 + 5n + 6}$$

divide through by the highest Power of n

$$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{n^2/n^2 + 3n/n^2 + n/n^2}{n^2/n^2 + 5n/n^2 + 6/n^2} = \frac{1+0+0}{1+0+0} = \frac{1}{1} = 1$$

It ~~may~~ could either be convergent or divergent using test 1 on U_n

$$U_n = 2/n^2 + 3n + 2$$

$$= \frac{2/n^2}{n^2/n^2 + 3n/n^2 + 2/n^2} = \frac{0}{1+0+0} = 0$$

Convergence

(b) $\frac{2}{1^2} + 2^2/2^2 + 2/3^2 + 2/4^2 + \cdots$

$$U_n = 2/n^2, \quad U_{n+1} = 2/(n+1)^2$$

$$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{2}{(n+1)^2} \times n^2/2$$

$$\frac{n^2}{n^2 + 2n + 1}$$

divide by the highest power of n

$$\frac{n^2/n^2}{n^2/n^2 + 2n/n^2 + 1/n^2} = \frac{1}{1+0+0} = 1/1$$ ~~pass~~

Could converge or diverge

Using test 1 on U_n

$$U_n = 2/n^2$$

$$\frac{2/n^2}{n^2/n^2} = 0/1 = 0$$

Converge

divide by the highest Power of n

$$\frac{n^2/n^2}{n^2/n^2 + 2n/n^2 + 1/n^2} = \frac{1}{1+0+0} = 1/1$$ ~~Fast~~

could converge or diverge

Using test 1 on U_n

$$U_n = 2/n^2$$

$$\frac{2/n^2}{n^2/n^2} = 0/1 = 0$$

Converge

c) $U_n = \dfrac{1+2n^2}{1+n^2}$

$$\lim_{n\to\infty} U_n = \frac{1/n^2 + 2n^2/n^2}{1/n^2 + n^2/n^2}$$

$$\frac{0+2}{0+1} = 2/1$$

$= 2$ diverge

③ $\frac{x}{27} + \frac{x^2}{125} + \dots \frac{+x^2}{(2n+1)^3}$

$U_n = \frac{x^n}{(2n+1)^3}$, $U_{n+1} = \frac{x^{n+1}}{(2(n+1)+1)^3} = \frac{x^{n+1}}{(2n+3)^3}$

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$

$\frac{x(2n+1)^3}{(2n+3)^3}$

$\frac{(8n^3+12n^2+6n+1)x}{8n^3+18n^2+54n+27}$

$\frac{8xn^3+12xn^2+6xn+x}{8n^3+18n^2+54n+27}$

divide by highest Power of n

$$\frac{8xn^3 + 12xn^2 + 6xn + x}{8n^3 + 18n^2 + 54n + 27}$$

divide by highest Power of n

$$\frac{8xn^3/n^3 + \frac{12xn^2}{n^3} + \frac{6xn}{n^3} + x/n^3}{\frac{8n^3}{n^3} + \frac{18n^2}{n^3} + \frac{54n}{n^3} + 27/n^3}$$

$$\frac{8x}{8}$$

$$\frac{8x}{8} < 1 \quad = \quad 8x < 8$$

$$x < 1$$

4) $\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3}\right)$ $\quad 8|x|<|8 \quad |x|<1 \quad$ $= \frac{\sin 0 - \cos 0}{0^3} = \frac{0-1}{0}$ undefi

$$\lim_{x \to 0} \frac{\cos 0 + \sin 0}{3(0)^2} = \frac{1+0}{0} \text{ undefined}$$

$$\lim_{x \to 0} = \frac{-\sin x + \cos x}{6x} = \frac{-\sin 0 + \cos 0}{6(0)} = \frac{-0+1}{0} \text{ undefined}$$

$$\lim_{x \to 0} \frac{-\cos x - \sin x}{6} = \frac{-\cos 0 - \sin 0}{6} = \frac{-1-0}{6} = -1/6$$