16/ENG02/019.

Computer Engineering.

ENGINEERING MATHEMATICS ASSIGNMENT.

Given: $x = \cos t + t\sin t$

$y = \sin t + t\cos t$

(1) Determine an expression for the radius of curvature (R).

(2) Expressions for the co-ordinates (h, k) of the centre of curvature.

Solutions:

(1) $x = \cos t + t\sin t$

$y = \sin t + t\cos t$

$x = \cos t + t\sin t$

$\frac{dx}{dt} = -\sin t + \sin t + t\cos t$

$\frac{\delta x}{\delta t} = t\cos t.$

$y = \sin t - t\cos t$

$\frac{\delta y}{\delta t} = \cos t - [-t\sin t + \cos t].$

$\frac{\delta y}{\delta t} = \cos t + t\sin t - \cos t$

$\frac{\delta y}{dt} = t\sin t$

$\frac{\delta y}{\delta x} = \frac{\delta y}{\delta t} \times \frac{\delta t}{\delta x}.$

$= \frac{t\sin t}{t\cos t}$

$\frac{\delta y}{\delta x} = \tan t.$

$\frac{d^2y}{\delta x^2} = \frac{\delta}{\delta x}\left(\frac{\delta y}{\delta x}\right)$

$= \frac{\delta}{\delta x}(\tan t).$

$= \sec^2 t.$

$=$

$$\sigma \frac{d^2y}{dx^2} = \frac{1}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}.$$

$$\sigma = \frac{\sec^2 t}{(1+\tan^2 t)^{3/2}}.$$

$$\sigma = \frac{1}{R}.$$

$$R = \underbrace{\left[1+\tan^2 t\right]^{3/2}}_{\sec^2 t}$$

Recall $1+\tan^2 t = \sec^2 t$

$$R = \frac{(\sec^2 t)^{3/2}}{\sec^2 t}$$

$$R = \frac{\sec^3 t}{\sec^2 t}$$

$$R = \sec t$$

$\therefore$ Radius of curvature is $\sec t$.

2) $h = x - R\sin\delta$

$k = y + R\cos\delta$. i.e $dy/dx = \tan\delta$.

$\tan\alpha$ extra $= \tan t$

$\tan\alpha$

$\alpha = \tan^{-1}(\tan t)$

$\alpha = t$

$h = \cos t + \sin t - \sec t \sin t$

$h = \cos t + \sin t - \tan t$

$k = (\sin t - \tan\cos t)$ reflect cost.

$k = \sin t - t\cos t$.

The co-ordinates $(h, k) = (\cos t + \sin t - t\tan t, \sin t - t\cos t)$.

$= (\cos t + \sin t - t\tan t, \sin t - t\cos t)$.