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161ENG05005
MECHATRONICS ENG 281

1) $x = \cos t + \sin t$

$y = \sin t - t\cos t$

Find An expression for the radius of curvature (R) in terms of t.

answer

$x = \cos t + t\sin t$

$$\frac{dx}{dt} = -\sin t + t\cos t + \sin t = t\cos t$$

$y = \sin t - t\cos t$

$$\frac{dy}{dt} = \cos t + t\sin t - \cos t = t\sin t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\Rightarrow \frac{\cancel{t}\sin t}{\cancel{t}\cos t}$$

$$\frac{dy}{dx} = \frac{\sin t}{\cos t}$$

$$\frac{d^2y}{dx^2} \Rightarrow u = \sin t \qquad v = \cos t$$

$$du/dt = \cos t \qquad dv/dt = -\sin t$$

$$\frac{v\,du/dt - u\,dv/dt}{v^2}$$

$$\frac{(\cos t)(\cos t) - (\sin t)(-\sin t)}{(\cos t)^2} \times \frac{1}{t\cos t}$$

$$= \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{1}{t\cos t}$$

$$\Rightarrow \frac{1}{\cos^2 t} \times \frac{1}{t\cos t} = \frac{1}{t\cos^2 t}$$

$$\frac{d^2y}{dx^2} = \frac{1}{t\cos^2 t}$$

$$R = \frac{\left(1+\left(dy/dx\right)^2\right)^{3/2}}{d^2y/dx^2}$$

$$R = \frac{\left[1+\left(\frac{\sin t}{\cos t}\right)^2\right]^{3/2}}{d^2y/dx^2}$$

$$R = \frac{\left(1 + \frac{\sin^2 t}{\cos^2 t}\right)^{3/2}}{\frac{1}{t\cos^3 t}}$$

$$= \frac{\left(1/\cos^2 t\right)^{3/2}}{1/t\cos^3 t}$$

$$= \frac{1/\cos^3 t}{1/t\cos^3 t}$$

$$R = \frac{1}{\cos^3 t} \times t\cos^3 t$$

b) Expression for the coordinates (h, k) of the centre of curvature in terms of t.

answer.

$$h = x_1 - R\sin\theta$$
$$k = y_1 + R\cos\theta$$

$$R = t$$
$$\theta \Rightarrow \tan^{-1}(dy/dx)$$

$$\frac{dy}{dx} \Rightarrow \frac{\sin t}{\cos t} = \tan t$$

$$\Rightarrow \quad \theta = \frac{1}{\tan} \times \frac{\tan t}{1}$$

$$\theta = t$$
$$h = x_1 - t\sin t$$
$$k = y_1 + t\cos t$$

But $x = \cos t + t\sin t$

$y = \sin t - t\cos t$

$$h = \cos t + t\sin t - t\sin t$$
$$h = \cos t$$

$$k \Rightarrow \sin t - t\cos t + t\cos t$$
$$= \sin t$$

$$(h, k) = (\cos t, \sin t)$$

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