Ibe Amarachi Sheila
Mechanical Engineering
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## Assignment

The parametric equations of a curve are as given in equations (1) and (2)

$x = \cos t + t\sin t$ ..... (1)

$y = \sin t - t\cos t$ ..... (2)

In terms of t, determine:

i) An expression for the radius of curvature (R), and

ii) Expressions for the coordinates (h, k) of the centre of curvature

## SOLUTION)

$x = \cos t + t\sin t$ ; $dx/dt = t\cos t$.

$y = \sin t - t\cos t$ ; $dy/dt = t\sin t$.

$dx/dt = -\sin t + t\cos t + \sin t$

$dx/dt = t\cos t$.

$dy/dt = \cos t + t\sin t - \cos t$

$dy/dt = t\sin t$.

$dy/dx = dy/dt \div dx/dt$.

$dy/dx = \frac{t\sin t}{t\cos t} = \frac{\sin t}{\cos t}$.

Let;

$u = \sin t$ ; $du/dt = \cos t$.

$v = \cos t$ ; $dv/dt = -\sin t$.

$du/dx = \cos t \times \left(\frac{1}{t\cos t}\right) = \frac{\cos t}{t\cos t}$

$dv/dx = -\sin t \times \left(\frac{1}{t\cos t}\right) = \frac{-\sin t}{t\cos t}$.

$$d^2y/dx^2 = \frac{v\,dy/dx - u\,dv/dx}{v^2}$$

$$d^2y/dx^2 = \frac{\cos t\left(\frac{\cos t}{t\cos t}\right) - (\sin t)\left(\frac{-\sin t}{t\cos t}\right)}{(\cos t)^2}$$

$$d^2y/dx^2 = \frac{\cos^2 t/t\cos t + \sin^2 t/t\cos t}{\cos^2 t}$$

$$d^2y/dx^2 = \frac{\cos^2 t + \sin^2 t}{t\cos t} \times \frac{1}{\cos^2 t}$$

$$d^2y/dx^2 = \frac{1}{t\cos^3 t}$$

$$R = \frac{[1 + (dy/dx)^2]^{3/2}}{d^2y/dx^2}$$

$$R = \left[1 + \left(\frac{\sin t}{\cos t}\right)^2\right]^{3/2} \div \frac{1}{t\cos^3 t}$$

$$R = \left[\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right]^{3/2} \times \frac{t\cos^3 t}{1}$$

$$R = \frac{1}{(\cos^2 t)^{3/2}} \times t\cos^3 t$$

$$R = \frac{t\cos^3 t}{\cos^3 t} = t.$$

$\therefore R = t.$

ii) $h = x_1 - R\sin\theta$

$K = y_1 + R\cos\theta$

$\theta = \tan^{-1}\left(dy/dx\right).$

$\theta = \tan^{-1}\left(\frac{\sin t}{\cos t}\right) = \tan^{-1}(\tan t).$

$\therefore \theta = t.$

$R = t.$

$h = \cos t + t\sin t - t\sin t$

$\therefore h = \cos t$.

$k = \sin t - t\cos t + t\cos t$

$\therefore k = \sin t$.