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16/MHS01/005
ELECTRICAL/ELECTRONICS ENGINEERING
ENG 281 (Engineering maths 1)

1 a) $\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4) \sin(\cos x)}{x - \pi/2} \right]$

$= \left[\frac{((\pi/2)^2 - \pi/8) \sin(\cos \pi/2)}{\pi/2 - \pi/2} \right] = \left[\frac{((\pi/2)^2 - \pi/8) \sin(\cos \pi/2)}{0} \right]$

Undefined

Using l'hopitals rule

$\cancel{du/dx} \Rightarrow \quad u = x^2 - \pi/4$

$v = \sin(\cos x)$

$\frac{du}{dx} = 2x \quad \frac{dv}{dx} = -\sin x \cos(\cos x)$

$u \, du/dx + v \, du/dx$

$\frac{(x^2 - \pi/4) \times -\sin x \cos(\cos x) + \sin(\cos x) 2x}{1}$

$= (\pi/2)^2 - \pi/4 \times -\sin 90 \cos(\cos 90) + \sin \cos 90 \times 2(\pi/2)$

$= \left(\frac{\pi^2}{4} - \pi/4\right) \times -1 + 0 \times \pi$

$= -\left(\frac{\pi^2}{4}\right) + \frac{\pi}{4} = \frac{\pi}{4} - \frac{\pi^2}{4}$

$\therefore \lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4) \sin(\cos x)}{x - \pi/2} \right] = \frac{\pi(1 - \pi)}{4}$

1b) $\lim_{x \to \pi/3} \ln\left[\exp\left(\frac{3x^2+2x-1}{x+1}\right)\right]$

$\lim_{x \to \pi/3} \ln\left[\exp \frac{(3x-1)(x+1)}{(x+1)}\right]$

$\lim_{x \to \pi/3} \ln\left[\exp(3x-1)\right]$

$\lim_{x \to \pi/3} 3x-1$

$3\left(\frac{\pi}{3}\right)-1$

$= \frac{3\pi}{3} - 1$

1c) $\lim_{x \to 2\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{x-\sqrt{3}}\right]$

$\cos\left[\sin^{-1}\frac{(2\sqrt{3}-2)}{2\sqrt{3}-\sqrt{3}}\right]$

$\cos\left[\sin^{-1}\frac{\sqrt{3}}{2}\right]$

$\cos[60] = 1/2$

1d) $\lim_{x \to 4}\left[\frac{x^2-8x+16}{x^2-5x+4}\right]$

$\frac{4^2-8(4)+16}{4^2-5(4)+4} = \frac{16-32+16}{16-20+4} = 0/0$ undefined

Using L'hopitals rule

$\frac{d}{dx}\left[\frac{x^2-8x+16}{x^2-5x+4}\right]$

$\lim_{x \to 4} \frac{2x-8}{2x-5}$ $\qquad \frac{2(4)-8}{2(4)-5}$

$= \frac{8-8}{8-5} = 0/3$

2a) $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \dots$

$U_n = \frac{2}{(n+1)\cdot(n+2)}$

$U_{n+1} = \frac{2}{(n+2)\cdot(n+3)}$

Using d'Alemberts ratio test

$$\left|\frac{U_{n+1}}{U_n}\right| = \frac{2}{(n+2)\cdot(n+3)} \div \frac{2}{(n+1)\cdot(n+2)}$$

$$= \frac{2}{(n+2)\cdot(n+3)} \times \frac{(n+1)\cdot(n+2)}{2}$$

$$= \frac{n+1}{n+3}$$

$$\lim_{n\to\infty} = \frac{0+1}{0+3} = 1/3 < 1$$

Convergence.

b) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \dots$

Using Comparison test. Recall!

$$\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots \frac{1}{n^2}\right] = \sum_{n=1}^{\infty} \cdot \frac{1}{n^2}$$

$$\Rightarrow \left[\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \dots \frac{2}{n^2}\right] = \sum^{\infty} \cdot \frac{2}{n^2}$$

$P = 2$

$P > 1$ Convergence

c) $U_n = \frac{1+2n^2}{n^2}$ $\quad U_{n+1} = \frac{1+(2n^2+1)^2}{(n+1)^2}$

Using d'Alemberts ratio

1+

2c) $U_n = \frac{1+2n^2}{1+n}$

$$U_{n+1} = \frac{1+2(n+1)^2}{1+(n+1)}$$

$$U_{n+1} = \frac{1+2[n^2+2n+1]}{2+n} = \frac{2n^2+4n+3}{2n}$$

$$\frac{2n^2+4n+3}{2+n} \div \frac{1+2n^3}{1+n}$$

$$\frac{2n^2+4n+3}{2+n} \times \frac{1+n}{1+2n^2}$$

$$\frac{(2n^2+4n+3)(1+n)}{(2+n)(1+2n^2)}$$

$$\lim_{n\to\infty}\left(\frac{U_{n+1}}{U_n}\right) = \lim_{n\to\infty}\left(\frac{2+6/n+7/n^2+3/n^2}{2+4/n+1/n^3+2/n^3}\right)$$

$$\lim_{n\to\infty} = \left(\frac{1+2n^3}{1+n}\right)$$

$= 1$ divergent

3

$U_n = \frac{x^n}{(2n+1)^3}$, $U_{n+1} = \frac{x^{n+1}}{(2n+2)^3}$

Using d'Alemberts ratio test

$\frac{U_{n+1}}{U_n}$ $\frac{x^{n+1}}{(2n+2)^3}$

$\frac{x(2n+1)^3}{(2n+2)^3} = \frac{8n^3 + 12n^2 + n + 1}{8n^3 + 24n^2 + 24n + 8}$

divide by n^3

$= \frac{(8 + {}^2/_n + {}^1/_{n^2} + {}^1/_{n^3})}{(8 + {}^{12}/_n + {}^{24}/_{n^2} + {}^8/_{n^3}}$

$8x/_8 \geq x - 1$ $x < 1$

4) $\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$

by using l'hopital's rule

$y = \frac{\sin x - \cos x}{x^3}$

$dy/_{dx} = \cancel{-\cos x}\ \frac{-\cos x - \sin x}{3x^2}$; $\frac{-\cos 0 - \sin(0)}{3(0)^2}$ = Undefined

$d^2y/_{dx^2} = \frac{-\sin x + \cos x}{6x}$; $\frac{-\sin 0 + \cos 0}{6(0)}$ Undefined

$d^3y/_{dx^3} = \frac{-\sin x + \cos x}{6}$: $\cancel{-\sin 0} + \frac{\cos 0 + \sin 0}{6}$

$= \frac{1}{6}$

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