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Assignment 2

1 $x = \cos t + t\sin t$

$y = \sin t - t\cos t$

An expression for the radius of curvature (R) in terms of t

Solo

$x = \cos t + t\sin t$

$\frac{dx}{dt} = -\sin t + t\cos t + \sin t = t\cdot\cos t$

$y = \sin t - t\cos t$

$\frac{dy}{dt} = \cos t + t\sin t - \cos t = t\sin t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{t\sin t}{t\cos t}$

$\frac{dy}{dx} = \frac{\sin t}{\cos t}$

$\frac{d^2y}{dx^2} = u = \sin t$

$v = \cos t$

$du/dt = \cos t$, $dv/dt = -\sin t$

$$\frac{v\,du/dt - u\,dv/dt}{v^2}$$

$$\frac{\cos t(\cos t) - (\sin t)(-\sin t)}{(\cos t)^2} \times \frac{1}{t\cos t}$$

$$\frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{1}{t\cos t}$$

$$\frac{1}{\cos t} \times \frac{1}{t\cos t} = \frac{1}{t\cos^2 t}$$

$$\frac{d^2y}{dx^2} = \frac{1}{t\cos^2 t}$$

$$R = \left(\frac{1 + (dy/dx)^2}{d^2y/dx^2}\right)^3$$

$$R = \frac{\left(1 + \left(\frac{\sin t}{\cos t}\right)^2\right)^{3/2}}{d^2y/dx^2}$$

$$R = \frac{\left(1 + \frac{\sin^2 t}{\cos^2 t}\right)^{3/2}}{1/t\cos^3 t}$$

$$= \frac{\left[\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right]^{3/2}}{1/t\cos^3 t}$$

$$R = \frac{\left(1/\cos^2 t\right)^{3/2}}{1/t\cos^3 t}$$

$$= \frac{1/\cos^3 t}{1/t\cos^3 t}$$

$$= \frac{1}{\cos^3 t} \times t\cos^3 t$$

$$R = t$$

2 Expression for the co-ordinates $[h, k]$ of the centre of curvature in terms of t

Soln

$$h = x_1 - R\sin\theta$$

$$k = y_1 + R\cos\theta$$

$$R = t$$

$$\theta = \tan^{-1}\left(\frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = \frac{\sin t}{\cos t} = \tan t$$

$$\theta = \frac{1}{\tan t} \times t\tan t$$

$$\theta = t$$

$$h = x_1 - t\sin t$$

$$k = y_1 - t\cos t$$

But $x = \cos t + t\sin t$

$y = \sin t - t\cos t$

$$h = \cos t + t\sin t - t\sin t$$

$$u = \cos t$$

$$v = \sin t - t\cos t + \cos t$$

$$v = \sin t$$

$$(u, v) = (\cos t, \sin t)$$