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15/Eng02/035
Assignment 381 (Maths)

ENG 381 Assignment I

Solve the following

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$, given that at $x = 0$, $y = 1$ and $\frac{dy}{dx} = -$

7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

Solution

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Auxillary equation: $m^2 - m - 2$

$\therefore GS = CF + PI$

CF: Solve LHS $= 0$ $\therefore m^2 - m - 2 = 0$

$\therefore (m^2 - 2m + m - 2) = 0$

$m(m-2) + 1(m-2)$ $\therefore m = 2$ or -1

$\therefore y = Ae^{-x} + Be^{2x}$

PI: $f(x) = 8$. Assume $y = C$

$\therefore \frac{dy}{dx} = 0$; $\frac{d^2y}{dx^2} = 0$

Substituting $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in the given equation

$0 - 0 - 2C = 8$

$\frac{-2C}{-2} = \frac{8}{-2}$

$C = -4$

$PI = -4$

$GS = CF + PI$

$\therefore y = Ae^{-x} + Be^{2x} - 4$

$\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$GS = CF + PI$

CF: Solve LHS $\therefore$ Auxillary equation $m^2 - 4 = 0$

$m^2 = 4 \quad \therefore m = \pm 2$

$\therefore y = A\cosh 2x + B\sinh 2x$

PI: $f(x) = 10e^{3x}$ $\therefore$ Assume $y = Ce^{3x}$

$dy/dx = 3Ce^{3x}$; $d^2y/dx^2 = 9Ce^{3x}$

Substitute d^2y/dx^2 and y in the given equation

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$5C = 10$

$C = 2 \qquad \therefore y = 2e^{3x}$

$GS = CF + PI$

$y = A\cosh 2x + B\sinh 2x + 2e^{3x}$

$d^2y/dx^2 + 2\,dy/dx + y = e^{-2x}$

$GS = CF + PI$

CF: Solve LHS=0 $\therefore$ Auxillary equation $m^2 + 2m + 1 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 - (4 \times 1 \times 1)}}{2 \times 1} = \frac{-2 \pm 0}{2}$

$m = -1$ twice

$y = e^{-x}(A + Bx)$

PI $\Rightarrow f(x) = e^{-2x}$ $\therefore$ Assume $y = Ce^{-2x}$

$dy/dx = -2Ce^{-2x}$; $d^2y/dx^2 = 4Ce^{-2x}$

Substitute the value of d^2y/dx^2 and dy/dx

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

$C = 1 \qquad \therefore y = e^{-2x}$

$GS = CF + PI$

$y = e^{-x}(A + Bx) + e^{-2x}$

4) $d^2y/dx^2 + 25y = 5x^2 + x$

$GS = CF + PI$

$CF \Rightarrow$ Solve $LHS = 0$ $\therefore$ Auxillary equation $m^2 + 25 = 0$

$m^2 = -25$ $\therefore m = \pm j5$

$\therefore y = A\cos 5x + B\sin 5x$

$PI: f(x) = 5x^2 + x$ $\therefore$ Assume $y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

Substituted d^2y/dx^2 and y in the equation

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

x^2: $25C = 5$

$C = 1/5$

x: $25D = 1$

$D = 1/25$

x^0: $2C + 25E = 0$

$2(1/5) + 25E = 0$

$25E = -2/5$

$E = -2/125$

$\therefore y = \left(\frac{1}{5} \times x^2\right) + \left(\frac{1}{25} \times x\right) + \left(-\frac{2}{125}\right)$

$y = \frac{x^2}{5} + \frac{x}{25} - \frac{2}{125}$

$y = \frac{25x^2 + 5x - 2}{125}$

$GS = CF + PI$

$GS = A\cos 5x + B\sin 5x + \frac{25x^2 + 5x - 2}{125}$

5) $d^2y/dx^2 - 2\,dy/dx + y = 4\sin x$

$GS = CF + PI$

$CF \Rightarrow$ Solve $LHS = 0$ $\therefore$ Auxillary equation $m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0 \Rightarrow m(m-1) - 1(m-1) = 0$

$m = -1$ twice

$y = e^{-x}(A + Bx)$

$PI \Rightarrow f(x) = 4\sin x$ $\therefore$ Assume $y = C\cos x + D\sin x$

$dy/dx = -C\sin x + D\cos x$

$d^2y/dx^2 = -C\cos x - D\sin x$

Substitute into the general equation

$-C\cos x - D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$(-D + 2C + D)\sin x + (-C - 2D + C)\cos x = 4\sin x$

$\sin x$: $-D + 2C + D = 4$

$\frac{2C}{2} = \frac{4}{2} \quad \therefore C = 2$

$\cos x$: $-C - 2D + C = 0$

$-2 - 2D + 2 = 0$

$-2D = 0$

$D = 0$

$y = 2\cos x + 0\sin x$

$y = 2\cos x$

$GS = CF + PI \quad \therefore GS = e^{-x}(A + Bx) + 2\cos x$

6) $d^2y/dx^2 + 4\,dy/dx + 5y = 2e^{-2x}$

CF $\Rightarrow$ Solve LHS = 0 $\quad m^2 + 4m + 5 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - (4 \times 1 \times 5)}}{2 \times 1}$

$= \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2i}{2} = -2 \pm j$

$y = e^{-2x}(A\cos x + B\sin x)$

PI $\Rightarrow f(x) = 2e^{-2x} \quad \therefore y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$, $\frac{d^2y}{dx^2} = 4Ce^{-2x}$

Substitute in the equation

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

e^{-2x}: $4C - 8C + 5C = 2$

$C = 2$

$y = 2e^{-2x}$

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

Question 6 Continuation:

$x = 0 \; ; \; y = 1$

$1 = A + 2$

$A = -1$

$y = e^{-2x}(-\cos x + B\sin x) + 2e^{-2x}$

$\frac{dy}{dx} = e^{-2x}(\sin x + B\cos x) - 2e^{-2x}(-\cos x + B\sin x) - 4e^{-2x}$

If $x = 0$ and $dy/dx = -2$

$-2 = B + 2 - 4$

$B = 0$

Particular solution is $y = e^{-2x}(-\cos x) + 2e^{-2x}$

$y = -e^{-2x}\cos x + 2e^{-2x}$

$y = e^{-2x}(2 - \cos x)$

7) $3\,d^2y/dx^2 - 2\,dy/dx - y = 2x - 3$

CF $\Rightarrow$ LHS $= 0$ $\therefore$ Auxillary equation $= 3m^2 - 2m - 1 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{2 \pm \sqrt{(-2)^2 - (4 \times 3 \times -1)}}{2 \times 1}$

$= \frac{2 \pm \sqrt{4 + 12}}{2} = \frac{2 \pm 4}{2}$

$m = 3$ or -1

$\therefore y = Ae^{-x} + Be^{3x}$

PI : $f(x) = 2x - 3$ Byt $y = Cx + D$

$\frac{dy}{dx} = C \; ; \; \frac{d^2y}{dx^2} = 0$

Substitute into the given equation

$3(0) - 2(C) - (Cx + D) = 2x - 3$

$-2C - Cx - D = 2x - 3$

$x: \quad -C = 2$

$C = -2$

$x^0: \quad -2C - D = -3$

$-2(-2) - D = -3$

$4 - D = -3$

$7 = D$

$\therefore y = -2x + 7$

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$GS = CF + PI$

$GS = Ae^{-x} + Be^{3x} - 2x + 7$

$d^2y/dx^2 - 6\,dy/dx + 8y = 8e^{4x}$

$m^2 - 6m + 8 = 0$ — (Auxillary equation)

$CF \Rightarrow LHS = 0$

$m^2 - 4m - 2m + 8 = 0$

$m(m-4) - 2(m-4) = 0$

$m = 4$ or 2

$y = Ae^{2x} + Be^{4x}$

$PI \Rightarrow f(x) = 8e^{4x}$ But $y = Ce^{4x}$

$\frac{dy}{dx} = 4Ce^{4x}$; $\frac{d^2y}{dx^2} = 16Ce^{4x}$

Substitute $\frac{d^2y}{dx^2}$ into the general equation

$16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$

$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 8e^{4x}$

$e^{4x}\div$ $16C - 24C + 8C = 8$