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ELECTRICAL/ELECTRONICS

the parametric equation of a curve is given
$x = \cos t + t\sin t$
$y = \sin t - t\cos t$

i) find the expression of the radius of curvature R
ii) Expression of centre of curvature ~~(h,k)~~ with cordinates (h,k)-

Solution

$x = \cos t + t\sin t$

i) $\frac{dx}{dt} = -\sin t + [t\cos t + \sin t]$

$= -\sin t + t\cos t + \sin t$

$= t\cos t$

~~Using product rule, $u = t$, v~~

$y = \sin t - t\cos t$

$\frac{dy}{dt} = \cos t - [t \cdot (-\sin t) + \cos t)$

$= \cos t + t\sin t - \cos t$

$= t\sin t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{t\sin t}{t\cos t}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left[\frac{t\sin t}{t\cos t}\right]$

$u = t\sin t \qquad v = t\cos t$

$du = t\cos t + \sin t \qquad dv = -t\sin t + \cos t$

$$\frac{d^2y}{dx^2} = \frac{v\,dy/dx - u\,dy/dx}{v^2}$$

$$= \frac{(t\cos t)\cdot(t\cos t + \sin t) - t\sin t\,(-t\sin t + \cos t)}{(t\cos t)^2}$$

$$= \frac{t^2\cos^2 t + t^2\sin^2 t}{(t\cos t)^2}$$

$$= \frac{t^2(\cos^2 t + \sin^2 t)}{t^2\cos^2 t}$$

$$\cos^2 t + \sin^2 t = 1$$

$$\frac{d^2y}{dx^2} = \frac{t^2}{t^2\cos t} = \frac{1}{\cos t}$$

Recall: $\frac{1}{R} = \frac{d^2y/dx^2}{\left[1 + \left[dy/dx\right]^2\right]^{3/2}}$

$$R = \left[1 + \left[\frac{t\sin t}{t\cos t}\right]^2\right]^{3/2} \div \frac{1}{\cos t}$$

$$= \left[\frac{1}{1} + \frac{t^2\sin^2 t}{t^2\cos^2 t}\right]^{3/2} \times \frac{\cos t}{1}$$

$$= \left[\frac{1}{1} + \frac{t^2(\sin^2 t)}{t^2(\cos^2 t)}\right]^{3/2} \times \cos t$$

$$= \left[1 + \frac{\sin^2 t}{\cos^2 t}\right]^{3/2} \times \cos t$$

$$= \frac{\cos^2 t + \sin^2 t}{\cos^2 t} = \left[1 + \tan^2 t\right]^{3/2} \times \cos t$$

$$= \left[\sqrt{1} + \sqrt{\tan^2 t}\right]^3 \times \cos t$$

$= (1 + \tan t)^3 \times \cos t$

$= (1 + \tan^3 t) \times \cos t$

$= \cos t + (\tan^3 t) \cos t$

$= \cos t + \frac{\sin^3 t}{\cos^3 t} \cos t$

$= \cos t + \frac{\sin^3 t}{\cos^2 t}$

$\frac{\cos^3 t + \sin^3 t}{\cos^2 t}$

$R = \frac{\cos^3 t + \sin^3 t}{\cos^2 t}$

ii) $\tan\theta = \frac{dy}{dx} = \frac{t \sin t}{t \cos t} = \tan t$

$\tan\theta = \tan t$

$\theta = t$

$x_1 = \cos t + t \sin t$

$y_1 = \sin t - t \cos t$

$h = \cos t + t \sin t - t \sin t$

$h = \cos t$

$k = \sin t - t \cos t + t \cos t$

$= \sin t$

Therefore; $h = \cos t$

$k = \sin t$