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Computer Engineering
16/ENG02/057
ENG 281 (Engineering mathematics I

1) $\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2} \right]$

for numerator

$u = x^2 - \pi/4$

$v = \sin(\cos x)$

$\frac{du}{dx} = 2x$

$\frac{dv}{dx}$ $y = \sin(\cos x)$

$y = \sin u$, $dy/du = \cos u$
$u = \cos x$, $du/dx = -\sin x$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\frac{dy}{dx} = \cos u \times -\sin x$; $\therefore \frac{dv}{dx} = \cos^2 x \cdot -\sin x = -\cos^2 x \sin x$

$u\frac{dv}{dx} + v\frac{du}{dx}$

$x^2 - \frac{\pi}{4} \cdot -\cos^2 x \sin x + \sin(\cos x) \cdot 2x$

$\frac{dz}{dx} = -x^2 - \frac{\pi}{4} \cdot \cos^2 x \sin x + 2x \sin(\cos x)$

denominator,

$x - \pi/2$

$\frac{dy}{dx} = 1$

$\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2} \right] = \lim_{x \to \pi/2} \left[\frac{-x^2 - \pi/4 \cdot \cos^2 x \sin x + 2x\sin(\cos x)}{1} \right]$

$= \frac{-(\pi/2)^2 - \pi/4 \cdot \cos \quad \cdot \cos(\pi/2) \sin(\pi/2) + 2(\pi/2)\sin[\cos(\pi/2)]}{1}$

$$\Rightarrow \frac{\pi^2/_4 - \pi/_4 \cdot 1 + 0}{1}$$

$$\Rightarrow \pi^2/_4 - \pi/_4 \,//$$

b) $\lim_{x \to \pi/2} \ln \left[\exp \left(\frac{3x^2 + 2x - 1}{x+1} \right) \right]$

$\lim_{x \to \pi/2} \ln \left[\exp \left(\frac{(3x-1)\cancel{(x+1)}}{\cancel{(x+1)}} \right) \right]$

$\lim_{x \to \pi/2} \left[3x - 1 \right]$

$3 \left[\frac{\pi}{2} \right] - 1$

c) $\lim_{x \to 2+\sqrt{3}} \cos \left(\sin^{-1} \frac{(x-2)}{(x-\sqrt{3})} \right)$

$\cos \left(\sin^{-1} \frac{(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})} \right)$

$\cos \left[\sin^{-1} \left[\frac{\sqrt{3}}{2} \right] \right]$

$\cos(60) = 1/_2$

d) $\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right]$

$\lim \frac{(4^2 - 8(4) + 16}{4^2 - 5(4) + 4} = \frac{16 - 32 + 16}{16 - 20 + 4} = \frac{0}{0} = 0$

is 0 μ

$\lim_{x \to 4} \left[\frac{2x - 8}{2x - 5} \right]$

$\frac{2(4) - 8}{2(4) - 5} = \frac{0}{3} = 0 \,//$

2) $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \dots$

soln

$\frac{2}{6} + \frac{2}{12} + \frac{2}{20} + \frac{2}{30} + \dots$

$r = \frac{\cancel{2}}{12} \times \frac{6}{\cancel{2}}$

$r = 1/2$

$a = 2/6 = 1/3$

$S_n = a\frac{(1-r^n)}{1-r}$

$S_n = \frac{1/3(1 - 1/2^n)}{1 - 1/2}$

$S_n = \frac{1/3(1 - 1/2^n)}{1/2}$

$n \to \infty,\ 1/2^n \to 0$

$S_n = \frac{1/3(1-0)}{1/2}$

$S_n = 1/3 \times 2/1$

$S_n = 2/3$

Since $2/3$ is definite, then it converges

b) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \dots$

$\frac{2}{1} + \frac{2}{4} + \frac{2}{9} + \frac{2}{16} + \dots$

$r = \frac{\cancel{2}}{4} \times \frac{1}{\cancel{2}}$

$r = 1/4$

$a = 2$

$S_n = a\frac{(1-r^n)}{1-r}$

$S_n = \frac{2(1 - 1/4^n)}{1 - 1/4}$

$S_n = \frac{2(1 - 1/4^n)}{3/4}$

$n \to \infty,\ 1/4^n \to 0$

$S_n = \frac{2(1-0)}{3/4}$

$S_n = \frac{2}{3/4}$

$S_n = 8/3$

Since $8/3$ is definite, then it converges

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Assignment

2c. $U_n = \frac{1+2n^2}{1+n^2}$

$n \to \infty$

$$U_n = \frac{\frac{1}{n^2} + \frac{2\cancel{n^2}}{\cancel{n^2}}}{\frac{1}{n^2} + \frac{\cancel{n^2}}{\cancel{n^2}}}$$

$n \to \infty$, $1/n^2 \to 0$

$$U_n = \frac{0+2}{0+1}$$

$U_n = 2$

The series is divergent since $U_n \neq 0$.

4) $\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$

$$\frac{\sin 0 - \cos 0}{0^3}$$

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3. $\frac{x}{27} + \frac{x^2}{125} + \dots + \frac{x^n}{(2n+1)^3}$

$$U_n = \frac{x^n}{(2n+1)^3}$$

$$U_{n+1} = \frac{x^{n+1}}{(2n+1)^3 + 1}$$

$$U_{n+1} = \frac{x^{n+1}}{[2(n+2)+1}$$

$$U_{n+1} = \frac{x^{n+1}}{[(2n+2)^3 + 1}$$

$$U_{n+1} = \frac{x^{n+1}}{(2n+3)^3}$$

$$U_{n+1} = \frac{x^{n+1}}{8n^3 + 36n^2 + 54n + 27}$$

$[2(n+2)+1$

$$u_{n+1} = \frac{x^{n+1}}{[(2n+2)+1}$$

$$u_{n+1} = \frac{x^{n+1}}{(2n+3)^3}$$

$$u_{n+1} = \frac{x^{n+1}}{8n^3+36n^2+54n+27}$$

3)

$$\frac{u_{n+1}}{u_n} = \frac{x^{n+1}}{8n^3+36n^2+54n+27} \times \frac{(2n+1)^3}{x^n}$$

$$= \frac{\cancel{x^n} \cdot x^1}{8n^3+36n^2+54n+27} \times \frac{8n^3+12n^2+6n+1}{\cancel{x^n}}$$

$$= x \times \frac{8n^3+12n^2+6n+1}{8n^3+36n^2+54n+27}$$

$$= x \left[\frac{8n^3+12n^2+6n+1}{8n^3+36n^2+54n+27}\right]$$

$$\lim_{n\to\infty} x \left[\frac{8n^3/n^3 + 12n^2/n^3 + 6n/n^3}{8n^3/n^3 + 36n^2/n^3 + 54n/n^3 + 27/n^3}\right]$$

$$\lim_{n\to\infty} x \left[\frac{8 + 12/n + 6/n^2}{8 + 36/n + 54/n^2 + 27/n^3}\right]$$

$$\lim_{n\to\infty} x \left[\frac{8+0+0}{8\ \ 0+0+0}\right]$$

$$\frac{u_{n+1}}{u_n} = x$$

~~it / diverges / ...~~ $-1 \le x\ 1$ (it converges)

4)

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$$

$$\frac{\sin 0 - \cos 0}{0^3}$$

$$\frac{0 - 1}{0} = \frac{-1}{0} \text{ (undefined)}$$

$$\lim_{x \to 0} \left[\frac{\cos x + \sin x}{3x^2} \right]$$

$$\lim_{x \to 0} \frac{1 + 0}{0} = \text{undefined}$$

$$\lim_{x \to 0} \left[\frac{\cancel{\cos x} - \sin x + \cos x}{6x} \right]$$

$$\frac{0 + 1}{0} = \text{undefined}$$

$$\lim_{x \to 0} \left[\frac{-\cos x - \sin x}{6} \right]$$

$$\frac{-1 - 0}{6} = \frac{-1}{6}$$