

Assignment 2

The parametric equations of the ~~curve~~ ^{curve} are given

$$x = \cos t + t \sin t$$

$$y = \sin t - t \cos t$$

① To find the radius of curvature (R) in terms of t

$$\frac{dx}{dt} = -\sin t + [t \cos t + \sin t]$$
$$= -\sin t + t \cos t + \sin t$$

$$\frac{dx}{dt} = t \cos t$$

$$\frac{dy}{dt} = \cos t - [t - \sin t + \cos t]$$
$$= \cos t + t \sin t - \cos t$$

$$\frac{dy}{dt} = t \sin t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{t \sin t}{t \cos t} \times \frac{1}{t \cos t}$$

$$\frac{dy}{dx} = \frac{t \sin t}{t \cos t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dt} \left[\frac{t \sin t}{t \cos t} \right] \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{(t \cos t)(t \cos t + \sin t) - (t \sin t)(t \sin t + \cos t)}{(t \cos t)^2}$$

$$= \frac{t^2 \cos^2 t + t \cos t \sin t + t^2 \sin^2 t - t \cos t \sin t - t \cos t}{(t \cos t)^2}$$

$$= \frac{t^2 \cos^2 t + t^2 \sin^2 t}{(t \cos t)^2} \times \frac{1}{t \cos t}$$

$$\frac{t^2 (\cos^2 t + \sin^2 t)}{t^2 \cos^2 t} \times \frac{1}{t \cos t}$$

Recall $\cos^2 t + \sin^2 t = 1$

$$\frac{1}{\cos^2 t} \times t$$
$$t \cos^2 t$$

$$\frac{d^2 y}{dx^2} = \frac{1}{t \cos^3 t}$$

$$R = \left[1 + \left[\frac{dy}{dx} \right]^2 \right]^{3/2}$$

$$\frac{d^2 y}{dx^2}$$

$$= \left[1 + \left[\frac{t \sin t}{t \cos t} \right]^2 \right]^{3/2}$$

$$\frac{d^2 y}{dx^2} = \frac{1}{t \cos^3 t}$$

$$= \left[\frac{1 + t^2 \sin^2 t}{t^2 \cos^2 t} \right]^{3/2} \times \frac{t \cos^3 t}{1}$$

$$= \left[\frac{t^2 \cos^2 t + t^2 \sin^2 t}{t^2 \cos^2 t} \right]^{3/2} \times \frac{t \cos^3 t}{1}$$

$$\left[\frac{1}{\cos^2 t} \right]^{3/2} \times t \cos^3 t$$

$$= \left[\frac{1}{\cos^2 t} \right]^{3/2} \times t \cos^3 t$$

$$= \left[\frac{\sqrt{1}}{\sqrt{\cos^2 t}} \right]^3 \times t \cos^3 t$$

$$= \frac{1^3}{\cos^3 t} \times t \cos^3 t$$

$$R = t$$

Therefore the radius of curvature in terms of t is

$$R = t$$

11 For the coordinates of the centre of curvature (h, k)

$$h = x - R \sin \theta$$

$$k = y + R \sin \theta$$

$$\tan \theta = \frac{dy}{dx} = \frac{x \sin t}{x \cos t} = \tan t$$

$$\theta = \tan^{-1}(\tan t)$$

$$\theta = t$$

$$x_1 = \cos t + t \sin t \quad \text{from eqn (1)}$$

$$y_1 = \sin t - t \cos t \quad \text{from eqn (2)}$$

$$h = \cos t + t \sin t - [t] [\sin t]$$

$$h = \cos t + t \sin t - t \sin t$$

$$h = \cos t$$

$$k = \sin t - t \cos t + [t] [\cos t]$$

$$k = \sin t - t \cos t + t \cos t$$

$$k = \sin t$$

Therefore the expressions in terms of t for the centre of curvature (h, k) are

$$h = \cos t \quad \text{and} \quad k = \sin t$$