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The parametric equation of a curve are given in equation (i) and (ii)

$x = \cos t + t\sin t$

$y = \sin t - t\cos t$

In terms of t Determine;

(i) An expression for the radius of curvature (R) and
(ii) An expression for the co-ordinates of the centre of curvature.

Solution

(i) Recall

(i) $\Rightarrow R = \dfrac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$

$x = \cos t + t\sin t$

$y = \sin t + (-t\cos t) \therefore y = \sin t - t\cos t$

$\frac{dx}{dt} = -\sin t + t\cos t + \sin t$ --- (i)

$\frac{dx}{dt} = t\cos t.$

$dy/dt = \cos t + t\sin t - \cos t$

$\frac{dy}{dt} = t\sin t.$

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$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{t\sin t}{t\cos t} \quad = \frac{\sin t}{\cos t}$$

$$\frac{d^2y}{dx} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{\sin t}{\cos t}\right) \times \frac{dt}{dx}$$

$$v\,du/dx \cdot u\frac{dv}{dx}$$

$$= \frac{v\,du/dt - u\,dv/dt}{v^2}$$

Let $u = \sin t$

$v = \cos t$

$$\frac{du}{dt} = \cos t \qquad \frac{dv}{dt} = -\sin t$$

$$\frac{d^2y}{dx^2} = \frac{\cos t(\cos t) - \sin t(-\sin t)}{(\cos t)^2} \times \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{1}{t\cos t}$$

From trigonometric identities,

$$\sin^2\theta + \cos^2\theta = 1$$

$$\therefore \frac{\partial^2 y}{dx^2} = \frac{1}{t\cos^2 t}$$

Since $R = \dfrac{\left[1 + \frac{dy}{dx}^{2}\right]^{3/2}}{d^2y/dx^2}$

$R = \left[1 + \left(\frac{\sin t}{\cos t}\right)\right]^{3/2} - \frac{d^2y}{dx^2}$

$R = \left(1 + \frac{\sin^2 t}{\cos^2 t}\right)^{3/2} \times t\cos^2 t$

$R = \left(\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right) \times \frac{t\cos^2 t}{1}$

$R = \frac{1}{(\cos^2 t)^{3/2}} + t\cos^2 t$

$R = \frac{t\cos^2 t}{\cos^2 t}$

$\therefore \underline{R = t}$

<u>Expression for radius of curvature is t.</u>

(ii) (h, k)

Recall $h = x_1 - R\sin\theta$ ---- (i)

$k = y_1 + R\cos\theta$ --- (2)

$R = t$; $\theta = t$

$x_1 = \cos t + t\sin t$

$y_1 = \sin t + \cos t$

Substituting θ_1, x_1, y_1 and R - equation (i) and (ii)

$h = \cos t + t\sin t - t\sin t$

$\therefore h = \cos t$

$k = \sin t - t\cos t + t\cos t$

$k = \sin t$.

Hence, the expression of the co-ordinates (h, k) of the centre is $(\cos t, \sin t)$