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Course ENG 281
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Maths Assignment

1a.) $\lim_{x\to x/2}\left[\frac{(x^2-\pi/4)\sin(\cos x)}{x-x/2}\right]$

Soln,,

Let; (Numerator)

$u=x^2-x/4$; $du/dx=2x$

$V=\sin(\cos x)$; $dv/dx=-\sin x\cos(\cos x)$

Let;

$a=\cos x$; $da/dx=-\sin x$

$v=\sin a$; $dv/dx=\cos a$

$$dy/dx=\frac{dv}{da}\cdot\frac{da}{dx}$$

$$\frac{dv}{dx}=\cos a\cdot(-\sin x)$$

$dy/dx=-\sin x\cos a=-\sin x\cos(\cos x)$

$dy/dx=u\,dv/dx+v\,du/dx$

$dy/dx=(x^2-\pi/4)(-\sin x\cos(\cos x))+\sin(\cos x)(2x)$

$dy/dx=(x^2-\pi/4)(-\cos(\cos x)\sin x]+\sin(\cos x)(2x)$

Denominator;

Let;

$m=x-x/2$; $dm/dx=1$

$\lim_{x\to\pi/2}\left[\frac{(x^2-\pi/4)(\sin(\cos x)}{x-x/2}\right]$

$\lim_{x\to\pi/2}\left[(x^2-\pi/4)-\cos(\cos x)\sin x+\sin(\cos x)(2x)\right]$

$=(x/2)^2-\pi/4)-\cos(\cos\pi/2)\sin(\pi/2)+\sin(\cos\pi/2)(2(\pi/2))]$

$=\frac{(\pi^2/4-\pi/4)(-1)+0}{1}=-\pi^2/4+\pi/4$

$$\cancel{= -x^2 +} = \frac{-\pi^2 + \pi}{4}$$

$$\therefore \lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)(\sin(\cos x))}{x - \pi/2} \right] = \frac{\pi^2 + \pi}{4}$$

1b.) $\lim_{x \to \pi/2} \ln \left[\exp \frac{(3x^2 + 2x - 1)}{x+1} \right]$

Soln

$$\cancel{\lim} = \ln \left[\exp \frac{(3(\pi/2)^2 + 2(\pi/2) - 1)}{\pi/2 + 1} \right]$$

$$= \frac{3x^2/4 + x - 1}{x/2 + 1}$$

$$= \frac{3x^2}{4} + \frac{x}{1} - \frac{1}{1} \div \frac{x}{2} + \frac{1}{1}$$

$$= \frac{3x^2 + 4x - 4}{\cancel{4}\,2} \times \frac{\cancel{2}\,1}{x+2}$$

$$= \frac{3x^2 + 4x - 4}{2(x+2)} = \frac{(3x-2)(x+2)}{2(x+2)} = \frac{3x - 2}{2}$$

$$= 3/2\, x - 1$$

$$\therefore \lim_{x \to \pi/2} \ln \left[\exp \frac{(3x^2 + 2x - 1)}{x+1} \right] = 3/2\, x - 1$$

1c.) $\lim_{x \to 2+\sqrt{3}} \cos \left[\sin^{-1} \frac{(x-2)}{(x-\sqrt{3})} \right]$

Soln

$$= \cos \left[\sin^{-1} \frac{(2+\sqrt{3} - 2)}{(2+\sqrt{3} - \sqrt{3})} \right]$$

$$= \cos \left[\sin^{-1} \sqrt{3}/2 \right]$$

$$= \cos 60 = 1/2$$

$$\therefore \lim_{x \to 2+\sqrt{3}} \cos \left\{ \sin^{-1} \frac{(x-2)}{(x-\sqrt{3})} \right\} = 1/2$$

1d.) $\lim_{x\to 4}\left(\frac{x^2-8x+16}{x^2-5x+4}\right)$

Soln.

$\lim_{x\to 4}\left(\frac{2x-8}{2x-5}\right)$

$= \frac{2(4)-8}{2(4)-5} = 0/3 = 0$

$\therefore \lim_{x\to 4}\left(\frac{x^2-8x+16}{x^2-5x+4}\right) = 0$

2a.) $\frac{2}{2\times 3} + \frac{2}{3\times 4} + \frac{2}{4\times 5} + \frac{2}{5\times 6} + ---$

Soln.

$U_n = \frac{2}{(n+1)(n+2)}$

$U_{n+1} = \frac{2}{(n+1+1)(n+1+2)} = \frac{2}{(n+2)(n+3)}$

$\lim_{n\to\infty}\frac{U_{n+1}}{U_n} = \frac{2}{(n+2)(n+2)} \div \frac{2}{(n+1)(n+2)}$

$= \frac{\cancel{2}}{\cancel{(n+2)}(n+3)} \times \frac{(n+1)\cancel{(n+2)}}{\cancel{2}}$

$= \frac{(n+1)}{(n+3)}$

$\lim_{n\to\infty}\frac{n+1}{n+3} = \frac{n/n + 1/n}{n/n + 3/n}$

$= \frac{1+1/n}{1+3/n}$

$n\to\infty;\ 1/n \to 0;\ 3n \to 0$

$= \frac{1+0}{1+0} = 1.$

$\lim_{n\to\infty} = \frac{U_{n+1}}{U_n} = 1$; the series is either divergent or convergent

$\lim_{n\to\infty} U_n \frac{2}{(n+1)(n+2)} = \frac{2/n}{n^2+3n+2}$

$$= \frac{2/n^2}{1 + 3/n + 2/n^2} = \frac{0}{1} = 0$$

$\therefore$ the series is convergent since $\lim_{n \to \infty} U_n = 0$

2b.) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$

Soln,,

$$\sum_{n=1}^{\infty} 2/n^p$$

$p = 2$

Since $p > 1$; $\therefore$ the series converges

2c.) $U_n + = \frac{1 + 2n^2}{1 + n^2}$

Soln

$\lim_{n \to \infty} U_n$

$$\lim_{n \to \infty} \frac{1 + 2n^2}{1 + n^2} = \frac{1/n^2 + 2}{1/n^2 + 1}$$

$n \to \infty$; $1/n^2 \to 0$

$$= \frac{0 + 2}{0 + 1} = 2$$

Since $\lim_{n \to \infty} U_n \neq 0$ $\therefore$ the series is divergent.

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3.) $\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3}$

Soln.,

$$U_n = \frac{x^n}{(2n+1)^3}$$

$$U_{n+1} = \frac{x^{n+1}}{(2(n+1)+1)^3} = \frac{x^{n+1}}{(2n+2+1)^3} = \frac{x^{n+1}}{(2n+3)^3}$$

$$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{\frac{x^{n+1}}{(2n+3)^3}}{\frac{x^n}{(2n+1)^3}}$$

$$= \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$$

$$= \frac{x(2n+1)^3}{(2n+3)^3}$$

$$\lim_{n\to\infty} \frac{x(2n+1)^3}{(2n+3)^3}$$

$$\lim_{n\to\infty} \frac{x(8n^3+12n^2+6n+1)}{(8n^3+36n^2+54n+27)}$$

$$\lim_{n\to\infty} = \frac{8xn^3+12xn^2+6xn+6}{8n^3+36n^2+54n+27}$$

$$= \frac{8x + 12/n + 6x/n^2 + x/n^3}{8 + 36/n + 54/n^2 + 27/n^3}$$

$n \to \infty$; $12x/n \to 0$, $6x/n^2 \to 0$, $x/n^3 \to 0$, $36/n \to 0$, $54/n^2 \to 0$, $27/n^3 \to 0$

$$= \frac{8x+0+0+0}{8+0+0+0} = x.$$

$$\lim_{n\to\infty} \frac{x(2n+1)^3}{(2n+3)^3} = x.$$

$-1 \leq x \leq 1$, $\therefore$ the series is Convergent

4. $\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3}\right]$

$\lim_{x \to 0} \left[\frac{\cos x + \sin x}{3x^2}\right]$

$\lim_{x \to 0} = \left[\frac{-\sin x + \cos x}{6x}\right]$

$\lim_{x \to 0} \left[\frac{-\cos x - \sin x}{6}\right]$

$= \frac{-\cos(0) - \sin(0)}{6} = \frac{-1-0}{6} = -1/6$

$\therefore \lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3}\right] = -1/6$