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15/ENG06/008
Mechanical Engineering

01. $\frac{\partial^2 y}{\partial x^2} - \frac{\partial y}{\partial x} - 2y = 8$

Assuming hormogenity

$\frac{\partial^2 y}{\partial x^2} - \frac{\partial y}{\partial x} - 2y = 0$

let $y = e^{kx}$; $\partial y/\partial x = ke^{kx}$, $\partial^2 y/\partial x^2 = k^2 e^{kx}$

$\therefore k^2 e^{kx} - ke^{kx} - 2e^{kx} = 0$

$e^{kx}(k^2 - k - 2) = 0$

$\therefore k^2 - k - 2 = 8$

Factorising by completing the square method

$k^2 - k = 2$

$k^2 - k + (-\frac{1}{2})^2 = 2 + \frac{1}{4}$

$(k - \frac{1}{2})^2 = \frac{9}{4}$

$k = \pm\sqrt{9/4} + \frac{1}{2}$

$\therefore k_1 = \frac{3}{2} + \frac{1}{2} = 2$; $k_2 = -\frac{3}{2} + \frac{1}{2} = -1$

but $y = C_1 e^{k_1 x} + C_2 e^{k_2 x}$

$\therefore y_{homogenous} = C_1 e^{2x} + C_2 e^{-x}$

Let $y_{non-homogenous} = A$

$\frac{\partial y}{\partial x} = 0$, $\frac{\partial^2 y}{\partial x^2} = 0$

$\therefore 0 - 0 - 2A = 8$

$\therefore A = -4$

Hence, $y_{nm} = -4$

but $y = y_{homo} + y_{non}$

$\therefore y = C_1 e^{2x} + C_2 e^{-x} - 4$.

02. $\frac{\partial^2 y}{\partial x^2} - 4y = 10e^{3x}$

Assuming homogenity

$\frac{\partial^2 y}{\partial x^2} - 4y = 0$

Let $y = e^{kx}$, $\partial^2 y / \partial x^2 = k^2 e^{kx}$

$k^2 e^{kx} - 4y = 0$

let $y = e^{kx}$, $\partial^2 y / \partial x^2 = k^2 e^{kx}$

$k^2 e^{kx} - 4e^{kx} = 0$

$\therefore k^2 = 4$; $k = \pm 2$

$\therefore -k_1 = 2 \quad k_2 = -2$

Hence, $y = C_1 \cosh 2x + C_2 \sinh 2x$

let $y_{on} = Ae^{3x}$

$\frac{\partial y}{\partial x} = 3Ae^{3x}$, $\frac{\partial^2 y}{\partial x^2} = 9Ae^{3x}$

$\therefore 9Ae^{3x} - 4Ae^{3x} = 10e^{3x}$

$5Ae^{3x} = 10e^{3x}$

$5A = 10$

$\therefore A = 2$

Hence, $y_{non} = 2e^{3x}$

but $y = y_{homo} + y_{non}$

$= C_1 \cosh 2x + C_2 \sinh 2x + 2e^{3x}$.

Q3) $\frac{\partial^2 y}{\partial x^2} + 2\frac{\partial y}{\partial x} + y = e^{-2x}$

Let $y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2 e^{kx}$

$k^2 e^{kx} + 2ke^{kx} + e^{kx} = 0$

$\therefore k^2 + 2k + 1 = 0$

By completing the squares

$k^2 + 2k = -1$

$k^2 + 2k + 1^2 = -1 + 1$

$(k+1)^2 = 0$

$k = -1$

but $y = C_1 e^{kx} + xC_2 e^{kx}$

$\therefore y_{homo} = C_1 e^{-x} + xC_2 e^{-x} = e^{-x}(C_1 + xC_2)$

let $y_{non} = A e^{-2x}$

$y' = -2Ae^{-2x}$, $y'' = 4Ae^{-2x}$

$\therefore 4Ae^{-2x} - 4Ae^{-2x} = e^{-2x} - Ae^{-2x}$

$\therefore A = 1$

Hence, $y_{non} = e^{-2x}$

but $y = y_{homo} + y_{non}$

$\therefore y = e^{-x}(C_1 + xC_2) + e^{-2x}$

04) $\frac{\partial^2 y}{\partial x^2} + 25y = 5x^2 + x$

Assuming $y = e^{kx}$, $y'' = k^2 e^{kx}$

$k^2 + 25 = 0$

$k = -5;$

$k_1 = 5 \qquad k_2 = -5$

$\therefore y = C_1 \cos 5x + C_2 \sin 5x$

Let $y_{non} = Ax^2 + Bx + C$

$y' = 2Ax + B, \quad y'' = 2A$

$\therefore 2A + 25Ax^2 + 25Bx + 25C = 5x^2 + x$

$2A + 25C = 0$ —— (1)

$25B = 1 \qquad \therefore B = 1/25$ —— (2)

$25A = 5 \qquad \therefore A = 1/5$ —— (3)

Putting eq (3) in (1)

$2(1/5) + 25C = 0$

$\therefore 25C = -2/5$

Hence, $C = -2/125$

$\therefore y_{non} = 1/5 x^2 + 1/25 x - 2/125$

but $y = y_{homo} + y_{non}$

$\therefore y = C_1 \cos 5x + C_2 \sin 5x + 1/5 x^2 + 1/25 x - 2/125$

05) $\frac{\partial^2 y}{\partial x^2} - 2\frac{\partial y}{\partial x} + y = 4 \sin x$

let $y = 2^{kx}$, $y' = ke^{kx}$, $y'' = k^2 e^{kx}$

$k^2 e^{kx} - 2ke^{kx} + e^{kx} = 0$ (Assuming homogenity)

$k^2 - 2k + 1 = 0$

By completing the squares.

$k^2 - 2k = -1$

$k^2 - 2k + (-1)^2 = -1 + 1$

$(k-1)^2 = 0$

$\therefore k = 1$

Hence, $y_{hom} = C_1 e^x + x C_2 e^x$

let $y_{non} = A\sin x + B\cos x$, $y' = A\cos x - B\sin x$,

$y'' = -A\sin x - B\cos x$

$\cancel{y} - \cancel{A\sin x} - \cancel{B\cos x} - 2A\cos x + 2B\sin x + \cancel{A\sin x} + \cancel{B\cos x} = 4\sin x$

collecting like terms $\cdot -2A = 0$ $\quad A = 0$

$2B = 4$, $B = 2$

$\therefore y_{non} = 2\cos x$

$y = y_{homo} + y_{non}$

Hence, $y = C_1 e^x + x C_2 e^x + 2\cos x$

$\frac{\partial^2 y}{\partial x^2} + 4\frac{\partial y}{\partial x} + 5y = 2e^{-2x}$ [given that at $x=0, y'=1$ and $y''=3$]

Assuming homogeneity, let $y = e^{kx}$, $y' = ke^{kx}$ and $y'' = k^2 e^{kx}$

$\therefore k^2 + 4k + 5 = 0$

By completing the squares

$k^2 + 4k = -5$

$k^2 + 4k + (2)^2 = -5 + 4$

$(k+2)^2 = -1$

$k + 2 = \pm\sqrt{-1}$

$\therefore k = \pm i - 2$; $k_1 = i - 2$, $k_2 = -i - 2$

but $y = C_1 e^{k_1 x} + C_2 e^{k_2 x}$

$= C_1 e^{(i-2)x} + C_2 e^{(-i-2)x}$

$= C_1 e^{ix} \cdot e^{2x} + C_2 e^{-ix} \cdot e^{2x}$

$= e^{2x}(C_1 e^{ix} + C_2 e^{-ix})$

$= e^{2x}(C_1 \cos x + C_2 \sin x)$

Let $y_{non} = A e^{-2x}$

$y' = -2Ae^{-2x}$, $y'' = 4Ae^{-2x}$

$\therefore 4Ae^{-2x} - 8Ae^{-2x} + 5Ae^{-2x} = 2e^{-2x}$

$Ae^{-2x} = 2e^{-2x}$

$\therefore A = 2$

Hence $y_{non} = 2e^{-2x}$

but $y = y_{homo} + y_{non}$

$$y = e^{2x}(c_1\cos x + c_2\sin x) + 2e^{-2x}$$

but at $x=0$, $y=1$, $y'=-2$

$$y = c_1\cos x\, e^{2x} + c_2\sin x\, e^{2x} + 2e^{-2x}$$

$$y' = c_1[\cos x \cdot 2e^{2x} + e^{2x} \cdot -\sin x) + c_2(\sin x \cdot 2e^{2x} + e^{2x}\cos x\} - 4e^{-2x}$$

$$\therefore y' = c_1(2e^{2x}\cos x - \sin x\, e^{2x}) + c_2(2e^{2x}\sin x + e^{2x}\cos x) - 4e^{-2x}.$$

$$y'' = c_1(3e^{2x}\cos x] + c_2[4e^{2x}\cos x + 3e^{2x}\sin x] + 8e^{-2x}.$$

Inputting the conditions at $x=0$

$$y = c_1\cos(0)e^{2(0)} + c_2\sin(0)e^{2(0)} + 2e^{-2(0)} = 1$$

$$c_1 + 2 = 1$$

$$\therefore c_1 = -1$$

$$y' = c_1[2e^{2(0)}\cos(0) - \sin(0)e^{2(0)}] + c_2(2e^{2(0)}\sin(0) + e^{2(0)}\cos(0)] - 4e^{-2(0)} = -2$$

$$2c_1 + c_2 - 4 = -2$$

$$2c_1 + c_2 = 2$$

but $c_1 = -1$

$$-2 + c_2 = 2$$

$$c_2 = 4$$

$$\therefore y = e^{2x}(-\cos x + 4\sin x) + 2e^{-2x}$$

07) $$3\frac{\partial^2 y}{\partial x^2} - 2\frac{\partial y}{\delta x} - y = 2x - 3$$

Assuming homogeneity, let $y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2e^{kx}$

$$3k^2e^{kx} - 2ke^{kx} - e^{kx} = 0$$

$$3k^2 - 2k - 1 = 0$$

$$k^2 - 2/3k - 1/2 = 0$$

$$k^2 - 2/3k = 1/3 \text{ (completing the square)}$$

$$k^2 - 2/3 + (-2/6)^2 = 1/3 + 4/36$$

$$(k - 1/3)^2 \cancel{\ast} = 4/9$$

$$k - 1/3 = \pm\sqrt{4/9}$$

$$\therefore k = \pm 2/3 + 1/3$$

$$k_1 = 2/3 + 1/3 = 1 \quad , k_2 = -2/3 + 1/3 = -1/3$$

but $y = C_1 e^{k_1 x} + C_2 e^{k_2 x}$

$y_{homo} = C_1 e^{x} + C_2 e^{-1/3 x}$

Let $y_{non} = Ax + B$

$y' = A \quad , \quad y'' = 0$

$\therefore -2A - Ax - B = 2x - 3$

$\therefore -2A - B = -3 \longrightarrow 1$

$\therefore \cancel{-2A - B =} -2(-2) - B = -3$

$4 - B = -3$

$\therefore B = 7$

$\therefore y_{non} = -2x + 7$

but $y = y_{homogenous} + y_{non\text{-}homogenous}$

$\therefore y = C_1 e^{x} + C_2 e^{1/3 x} + 7 - 2x$

08 . $d^2y/dx^2 - 6\, dy/dx + 8y = 8e^{4x}$

Assuming homogenity, let $y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2 e^{kx}$

Hence,

$k^2 - 6k + 8 = 0$

By completing the square

$k^2 - 6k + (-3)^2 = -8 + 9$

$(k-3)^2 = 1$

$k - 3 = \pm\sqrt{1}$

$\therefore k = \pm 1 + 3$

$\therefore k = \pm 1 + 3$

$\therefore k_1 = 4, k_2 = 2$

but $y = C_1 e^{k_1 x} + C_2 e^{k_2 x}$

$\therefore y = C_1 e^{4x} + C_2 e^{2x}$

let $y_{non} = \cancel{Ax e^{4x}}\ Axe^{4x}$

$y' = A(4x e^{4x} + e^{4x})$

$y'' = A(\cancel{16e^{4x}}\ 16xe^{4x} + 4e^{4x} + 4e^{4x})$

$= A(16xe^{4x} + 8e^{4x})$

$2Ae^{4x} = 8e^{4x}$

Dividing both sides by $2e^{4x}$

$\therefore A = 4$

Hence, $y_{non} = 4x\, e^{4x}$

but $y = y_{homo} + y_{non}$

$\therefore y = C_1 e^{4x} + C_2 e^{2x} + 4xe^{4x}$

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