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15/ENG06/010
Mechanical Engineering

Assignment 1

1 $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$m^2 - m - 2 = 0$

$m^2 - 2m + m - 2$

$m(m-2) + 1(m-2)$

$(m+1)(m-2)$

$\therefore m_1 = -1 \quad , \quad m_2 = 2$

CF : $y = Ae^{-x} + Be^{2x}$

PI : $y = c$

$dy/dx = 0$

$\frac{d^2y}{dx^2} = 0$

$\therefore 0 - 0 - 2(c) = 8$

$-2c = 8$

$c = \frac{8}{-2} = -4$

GS $:\Rightarrow y = Ae^{-x} + Be^{2x} - 4.$

2. $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm\sqrt{4}$

$m = \pm 2$

CF: $y = A\cosh 2x + B\sinh 2x$

PI: $y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

$9ce^{3x} - 4ce^{3x} = 10e^{3x}$

$5ce^{3x} = 10e^{3x}$

$5c = 10$

$c = \frac{10}{5} = 2$

G.S: $y = A\cosh 2x + B\sinh 2x + 2e^{3x}$.

3. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

$\cancel{m^2 + 2m + 1 = 0}$ let $y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2e^{kx}$

$k^2e^{kx} + 2ke^{kx} + e^{kx} = 0$

$\therefore k^2 + 2k + 1 = 0$

By completing the squares,

$k^2 + 2k = -1$

$k^2 + 2k + 1^2 = -1 + 1$

$(k+1)^2 = 0$

$\therefore k = -1$

but $y = C_1e^{kx} + xC_2e^{kx}$

$\therefore y_{homo} = C_1e^{-x} + xC_2e^{-x} = e^{-x}(C_1 + xC_2)$

let $y_{non} = Ae^{-2x}$

$y' = -2Ae^{-2x}$, $y'' = 4Ae^{-2x}$

$\therefore 4Ae^{-2x} - 4Ae^{-2x} = e^{-2x} - Ae^{-2x}$

$\therefore A = 1$

Hence, $y_{non} = e^{-2x}$

but $y = y_{homo} + y_{non}$

$\therefore y = e^{-x}(C_1 + xC_2) + e^{-2x}$.

4. $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

Assuming $y = e^{kx}$, $y'' = k^2e^{kx}$

$k^2 + 25 = 0$

$\therefore k = \pm 5i$

$k_1 = 5i$, $k_2 = -5i$

$\therefore y_{homo} = C_1 \cos 5x + C_2 \sin 5x$

let $y_{non} = Ax^2 + Bx + c$

$y' = 2Ax + B$, $y'' = 2A$

$\therefore 2A + 25A x^2 + 25Bx + 25c = 5x^2 + x$

$2A + 25C = 0$ .... (1)

$25B = 1$, $\therefore B = \frac{1}{25}$ ...... (2)

$25A = 5$, $\therefore A = \frac{1}{5}$ ...... (3)

putting eqn (3) in (1)

$2\left(\frac{1}{5}\right) + 25C = 0$

$\therefore 25C = \frac{-2}{5}$

Hence, $C = \frac{-2}{125}$

$\therefore y_{non} = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

but $y = y_{homo} + y_{non}$

$\therefore y = C_1 \cos 5x + C_2 \sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$.

5. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

let $y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2 e^{kx}$

$k^2 e^{kx} - 2ke^{kx} + e^{kx} = 0$ (assuming homogeneity).

$k^2 - 2k + 1 = 0$

By completing the squares,

$k^2 - 2k = -1$

$k^2 - 2k + (-1)^2 = -1 + 1$

$(k-1)^2 = 0$

$\therefore k = 1$

Hence, $y_{homo} = C_1 e^x + xC_2 e^x$

let $y_{non} = A\sin x + B\cos x$, $y' = A\cos x - B\sin x$, $y'' = -A\sin x - B\cos x$

$y = -A\sin x - B\cos x - 2A\cos x + 2B\sin x + A\sin x + B\cos x = 4\sin x$.

Collecting like terms $-2A = 0$, $A = 0$

$2B = 4$, $B = 2$

$\therefore y_{non} = 2\cos x$

but $y = y_{homo} + y_{non}$

Hence, $y = C_1 e^{x} + xC_2 e^{x} + 2\cos x$.

6. $\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 5y = 2e^{-2x}$ (given that at $x=0$, $y = 1$ and $y' = -2$)

Assuming homogeneity, let $y = e^{kx}$, $y' = ke^{kx}$ and $y'' = k^2e^{kx}$.

$\therefore k^2 + 4k + 5 = 0$

By completing the squares,

$k^2 + 4k = -5$

$k^2 + 4k + (+2)^2 = -5 + 4$

$(k+2)^2 = -1$

$k + 2 = \pm\sqrt{-1}$

$\therefore k = \pm i - 2$; $k_1 = i - 2$, $k_2 = -i - 2$.

but $y = C_1 e^{k_1 x} + C_2 e^{k_2 x}$

$= C_1 e^{(i-2)x} + C_2 e^{(-i-2)x}$

$= C_1 e^{ix} \cdot e^{2x} + C_2 e^{-ix} \cdot e^{2x}$

$= e^{2x}(C_1 e^{ix} + C_2 e^{-ix})$

$y_{homo} = e^{2x}(C_1 \cos x + C_2 \sin x)$

let $y_{non} = Ae^{-2x}$

$y' = -2Ae^{-2x}$, $y'' = 4Ae^{-2x}$

$\therefore y = 4Ae^{-2x} - 8Ae^{-2x} + 5Ae^{-2x} = 2e^{-2x}$

$Ae^{-2x} = 2e^{-2x}$

$\therefore A = 2$

Hence, $y_{non} = 2e^{-2x}$

but $y = y_{homo} + y_{non}$

$y = e^{2x}(C_1 \cos x + C_2 \sin x) + 2e^{-2x}$

but at $x = 0$, $y = 1$, $y' = -2$

$y = C_1 \cos x e^{2x} + C_2 \sin x e^{2x} + 2e^{-2x}$

$y' = C_1 [\cos x \cdot 2e^{2x} + e^{2x} \cdot -\sin x] + C_2 [\sin x \cdot 2e^{2x} + e^{2x} \cdot \cos x] - 4e^{-2x}$

$\therefore y' = C_1 [2e^{2x}\cos x - \sin x e^{2x}] + C_2 [2e^{2x} \sin x + e^{2x}\cos x] - 4e^{-2x}$.

$y'' = C_1 [3e^{2x} \cos 2x] + C_2 [4e^{2x} \cos x + 3e^{2x} \sin x] + 8e^{-2x}$

Inputting the conditions at $x = 0$

$$y = C_1 \cos(0) e^{2(0)} + C_2 \sin(0) e^{2(0)} + 2e^{-2(0)} = 1$$

$$C_1 + 2 = 1$$

$$\therefore C_1 = -1$$

$$y' = C_1\left[2e^{2(0)}\cos(0) - \sin(0)e^{2(0)}\right] + C_2\left[2e^{2(0)}\sin(0) + e^{2(0)}\cos(0)\right] - 4e^{-2(0)} = -2$$

$$2C_1 + C_2 - 4 = -2$$

$$2C_1 + C_2 = 2$$

but, $C_1 = -1$

$$\therefore -2 + C_2 = 2$$

$$C_2 = 4$$

$$\therefore y = e^{2x}\left[-\cos x + 4\sin x\right] + 2e^{-2x}$$

7. $$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$$

Assuming homogeneity, let $y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2e^{kx}$

$$3k^2e^{kx} - 2ke^{kx} - e^{kx} = 0$$

$$3k^2 - 2k - 1 = 0$$

$$k^2 - \frac{2}{3}k - \frac{1}{3} = 0$$

$$k^2 - \frac{2}{3}k = \frac{1}{3} \quad \text{(Completing the square)}$$

$$k^2 - \frac{2}{3} + \left(-\frac{2}{6}\right)^2 = \frac{1}{3} + \frac{4}{36}$$

$$\left(k - \frac{1}{3}\right)^2 = \frac{4}{9}$$

$$k - \frac{1}{3} = \pm\sqrt{\frac{4}{9}}$$

$$\therefore k = \pm\frac{2}{3} + \frac{1}{3}$$

$$k_1 = \frac{2}{3} + \frac{1}{3} = 1, \quad k_2 = -\frac{2}{3} + \frac{1}{3} = \frac{-1}{3}$$

but $y = C_1e^{k_1x} + C_2e^{k_2x}$

$$y_{homo} = C_1e^{x} + C_2e^{-\frac{1}{3}x}$$

let $y_{non} = Ax + B$

$$y' = A, \quad y'' = 0$$

$$\therefore -2A - Ax - B = 2x - 3$$

$$\therefore -2A - B = -3 \quad \text{.......... (1)}$$

$$-A = 2, \quad A = -2 \quad \text{.......... (2)}$$

$-2(-2) - B = -3$

$4 - B = -3$

$\therefore B = 7$

$y_{non} = -2x + 7$

but $y = y_{homo} + y_{non-homo}$

$\therefore y = C_1e^{x} + C_2e^{-\frac{1}{3}x} + 7 - 2x$

8. $\frac{d^2y}{dx^2} - \frac{6dy}{dx} + 8y = 8e^{4x}$

Assuming homogeneity, let $y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2e^{kx}$.

Hence,

$k^2 - 6k + 8 = 0$

By Completing the square,

$k^2 - 6k + (-3)^2 = -8 + 9$

$(k-3)^2 = 1$

$k - 3 = \pm\sqrt{1}$

$\therefore k = \pm 1 + 3$

$\therefore k_1 = 4, \; k_2 = 2$

but $y = C_1e^{k_1x} + C_2e^{k_2x}$

$\therefore y_{homo} = C_1e^{4x} + C_2e^{2x}$

let $y_{non} = Axe^{4x}$

$y' = A(4xe^{4x} + e^{4x})$

$y'' = A(16xe^{4x} + 4e^{4x} + 4e^{4x})$.

$= A(16xe^{4x} + 8e^{4x})$

$\therefore 16Axe^{4x} + 8Ae^{4x} - 24Axe^{4x} - 6Ae^{4x} + 8Axe^{4x} = 8e^{4x}$.

$2Ae^{4x} = 8e^{4x}$

Dividing both sides by $2e^{4x}$.

$\therefore A = 4$

Hence, $y_{non} = 4xe^{4x}$

but $y = y_{homo} + y_{non}$.

$\therefore y = C_1e^{4x} + C_2e^{2x} + 4xe^{4x}$.