

AKERELER ÖZLİMİZİ.Ç ENG 281  
16/MHS01/021

$$x = \cos t + \sin t$$

$$y = \sin t - \cos t$$

$$\frac{dx}{dt} = -\sin t + (t \cos t + \sin t (1))$$

$$\frac{dx}{dt} = -\sin t + t \cos t + \sin t$$

$$\frac{dx}{dt} = t \cos t$$

$$\frac{dy}{dt} = \cos t + (-t \sin t + \cos t (1))$$

$$\frac{dy}{dt} = \cos t + \sin t - t \sin t$$

$$\therefore \frac{dy}{dt} = t \sin t$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{\cancel{t} \sin t}{\cancel{t} \cos t} = \frac{\sin t}{\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left( \frac{dy}{dx} \right) = \frac{d}{dt} \left( \frac{dy}{dx} \right) \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt} \left( \frac{\sin t}{\cos t} \right) \cdot \frac{dt}{dx}$$

$$= \frac{d}{dt} \frac{dy}{dx} - U \frac{dv}{dx}$$

$$= \frac{d}{dt} \frac{dy}{dx} - U \frac{dy}{dt}$$

where  $u = \sin t$ ,  $v = \cos t$ .

$$\frac{dy}{dt} = +\cos t, \quad \frac{dv}{dt} = -\sin t$$

$$\frac{d^2y}{dx^2} = \frac{\cos t (\cos t) - \sin t (-\sin t)}{\cos^2 t}$$

$$= \frac{\cos^2 t + \sin^2 t}{\cos^2 t}$$

Recall  $\cos^2 t + \sin^2 t = 1$

$$\therefore \frac{d^2y}{dx^2} = \frac{1}{\cos^2 t} \times \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{1}{\cos^2 t} \times \frac{1}{t \cos t} = \frac{1}{t \cos^3 t}$$

$$\text{Recall } R = \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{3/2} \cdot \frac{d^2y}{dx^2}$$

$$R = \left[ 1 + \left( \frac{\sin t}{\cos t} \right)^2 \right]^{3/2} \cdot \frac{d^2y}{dx^2}$$

$$R = \left[ \frac{1 + \sin^2 t}{\cos^2 t} \right]^{3/2} \times \frac{1}{\cos^3 t}$$

$$R = \left[ \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \right]^{3/2} \times \frac{1}{\cos^3 t}$$

$$R = \left( \frac{1}{\cos^2 t} \right)^{3/2} \times \frac{1}{\cos^3 t} = \frac{1}{(\cos t)^{2 \times 3/2}} \times \frac{1}{\cos^3 t}$$

$$R = \frac{1}{\cos^3 t} \times \frac{1}{\cos^3 t} = \frac{1}{\cos^6 t} \quad \therefore R = t$$

For the coordinate  $(h, k)$  of the centre of curvature, recall  $h = x - R \sin \theta$   
 $k = y + R \cos \theta$

where  $\theta = t$ .

$$\theta = \tan^{-1} \left[ \frac{dy}{dx} \right] = \tan^{-1} (\tan t) = t$$

$$\therefore h = \cos t - R \sin t = \cos t - t \sin t$$

$$h = \cos t$$

$$k = \sin t - R \cos t + t \cos t$$

$$k = \sin t$$

The co-ordinates for the Centre of Curvature are  $(\cos t, \sin t)$

$$\therefore I_{yy} = 0$$

$$\text{Similarly, } I_{xx} = \int \rho^2 dy^2 = \int \rho^2 dx^2$$

Moment of Inertia to Simple Area  
 Moment of Inertia  
 $I = \frac{bd^3}{12}$   $I_{xx} = \frac{bd^3}{12}$