MORE

15/EN/GO4/1059

UYAEGD EBUBE

ELECT/ELECT.

1. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

// convert equation into an homogeneus equation

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

$m^2 - m \quad 2 = 0$

$(m^2 + m)(-2m - 2) = 0$

$m(m+1) - 2(m+1) = 0$

$(m+1)(m-2) = 0$

$m_1 = -1 \quad m_2 = 2$

$y = Ae^{-x} + Be^{2x}$ // complementary function

$y = C$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

$0 - 0 - 2C = 8$

$-2C = 8$

$C = \frac{8}{-2} = -4$

$C = -4$ // particular integral.

$GS = Ae^{-x} + Be^{2x} - 4$

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2. $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$\frac{d^2y}{dx^2} - 4y = 0$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \sqrt{4}$

$m = \pm 2$

$y = C\cosh 2x + D\sinh 2x$ // complementary function

$y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$ $\quad \frac{d^2y}{dx^2} = 9Ce^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

// divide equation by e^{3x}

$5C = 10$

$C = \frac{10^2}{5_1} = 2$

$C = 2e^{3x}$ // particular integral

$G.S. = C\cosh 2x + D\sinh 2x + 2e^{3x}$

5. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + (m+1) = 0$

$(m+1)$ twice

$y = e^{-x}(A + Bx)$ // complementary function

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$ $\quad \frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} + -4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

// divide equation by e^{-2x}

$C = 1$

$C = e^{-2x}$ // particular integral

$G{\cdot}S = e^{-x}(A+Bx) + e^{-2x}$

4. $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

//convert equation into an homogeneous equation

$\frac{d^2y}{dx^2} + 25y = 0$

$m^2 + 25 = 0$

$m^2 = -25$

$m = \pm j5$

$y = C\cos 5x + D\sin 5x$ //complementary function

$y = Cx^2 + Dx + E \qquad dy/dx = 2Cx + D \qquad d^2y/dx^2 = 2C$

$2C + 25[Cx^2 + Dx + E] = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25C = 5$

$C = 5/25 = 1/5$

$25D = 1$

$D = 1/25$

$2C + 25E = 0$

$2[1/5] + 25E = 0$

$2/5 + 25E = 0$

$25E = -2/5$

$E = -2/5 \times 1/25 = -2/125$

$y = 1/5\,x^2 + 1/25\,x - 2/125$ //particular integral

$G.S. = C\cos 5x + D\sin 5x + 1/5\,x^2 + 1/25\,x - 2/125$

5 $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

//convert equation to an homogeneous equation

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$(m-1)$ (twice)

$m = 1$

$y = e^x(A + Bx)$ // particular integral

$y = C\cos x + D\sin x$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$-C\cos x - D\sin x - 2[-C\sin x + D\cos x] + C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$-C\cos x - 2D\cos x + C\cos x - D\sin x + 2C\sin x + D\sin x = 4\sin x$

$\cos x(-C - 2D + C) + \sin x(-D + 2C + D) = 4\sin x$

$-C - 2D + C = 0 \Rightarrow -2D = 0 \quad D = 0$

$-D + 2C + D = 4$

$2C = 4 \Rightarrow C = \frac{4}{2} = 2$

$y = 2\cos x + 0\sin x = 2\cos x$ // particular integral

G.S. $= e^x(A + Bx) + 2\cos x$

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$ given that $x = 0$, $y = 1$ and $\frac{dy}{dx} = -2$

Convert equation to an homogeneous equation

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$

$m^2 + 4m + 5 = 0$

$a = 1 \quad b = 4 \quad c = 5$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 5}}{2 \times 1}$

$m = \frac{-4 \pm \sqrt{16 - 20}}{2}$

$m = \frac{-4 \pm \sqrt{-4}}{2}$

$m = \frac{-4 \pm j2}{2}$

$m = -2 \pm j$

$m_1 = -2 + j \quad$ or $\quad m_2 = -2 - j$

$y = e^{-2x}(C\cos x + D\sin x)$ // particular integral

$y = \cancel{Cxe^{-2x}} \quad Ce^{-2x}$

$\frac{dy}{dx} = \cancel{-Cxe^{-2x} + Cx} \quad -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$\cancel{4Ce^{-2x}} + 4[2Ce^{-2x}] + Ce$

$4Ce^{-2x} + 4[-2Ce^{-2x}] + 5[Ce^{-2x}] = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$4C - 8C + 5C = 2$

$C = 2$

$y = 2e^{-2x}$ // particular integral

$y = e^{-2x}(C\cos x + D\sin x) + 2e^{-2x}$

7. $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

// convert equation into an homogeneous equation

$3m^2 - 2m - 1 = 0$

$a = 3 \quad b = -2 \quad c = -1$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 3 \times -1}}{2 \times 6}$

$m = \frac{2 \pm \sqrt{4 + 12}}{12}$

$m = \frac{2 \pm \sqrt{16}}{12}$

~~m = 1±~~ $m = \frac{2 \pm 4}{12} \Rightarrow m = \frac{1 \pm 2}{6}$

$m = \frac{1+2}{6} = \frac{3}{6} = \frac{1}{3}$ or $m = \frac{1-2}{6} = \frac{-1}{6}$

$m_1 = \frac{1}{3}$ or $m_2 = -\frac{1}{6}$

$y = Ae^{1/3} + Be^{-1/6}$

$y = Cx + D$

$\frac{dy}{dx} = C$

$\frac{d^2y}{dx} = 0$

$3(0) - 2C - (Cx + D) = 2x - 3$

$-2C - Cx - D = 2x - 3$

$-C = 2$

$C = -2$

$-2C - D = -3$

$-2(-2) - D = -3$

$4 - D = -3$

$D = 4 + 3 = 7 \qquad y = -2x + 7$

$G.S = Ae^{1/3} + Be^{-1/6} - 2x + 7$

8. $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

// convert equation into a homogeneous equation

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$

$m^2 - 6m + 8 = 0$

$m^2 - 2m - 4m + 8 = 0$

$m(m-2) - 4(m-2) = 0$

$(m-2)(m-4) = 0$

$m_1 = 2 \quad m_2 = 4$

$y = Ae^{2x} + Be^{4x}$ // complementary function

$y = Cxe^{4x}$

$\frac{dy}{dx} = 4Cxe^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$\cancel{16Ce^{4x}} + 8Ce^{4x} + 4Cxe^{4x} + Ce^{4x} + Cxe^{4x} = 8e^{4x}$

$16\cancel{Cxe^{4x} + 4Cxe^{4x}} + Cxe^{4x} + 8Ce^{4x} + Ce^{4x} = 8e^{4x}$

$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 6(4Cxe^{4x} + Ce^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$

$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$

$16Cxe^{4x} - 24Cxe^{4x} + 8Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 6Ce^{4x} = 8e^{4x}$

$2Ce^{4x} = 8e^{4x}$

$2C = 8$

$C = 8/2 = 4$

$y = 4xe^{4x}$ // particular integral

$G.S. = Ae^{2x} + Be^{4x} + 4xe^{4x}$