

$$x = \cos t + b \sin t$$

$$i) \frac{dx}{dt} = -\sin t + [t \cos t + \sin t]$$

$$= -\sin t + t \cos t + \sin t$$

$$= t \cos t$$

~~Using product rule,  $u = b$ ,  $v = t$~~

$$y = \sin t - t \cos t$$

$$\frac{dy}{dt} = \cos t - [t(-\sin t) + \cos t]$$

$$= \cos t + b \sin t - \cos t$$

$$= b \sin t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{b \sin t}{t \cos t}$$

$$\frac{d^2 y}{dx^2} = \frac{d}{dx} \left( \frac{dy}{dx} \right) = \frac{d}{dx} \left[ \frac{t \sin t}{t \cos t} \right]$$

$$u = b \sin t \quad v = t \cos t$$

$$du = t \cos t + \sin t$$

$$dv = -b \sin t + \cos t$$

$$\frac{d^2 y}{dx^2} = \frac{v \frac{dv}{dx} - u \frac{du}{dx}}{v^3}$$

$$= \frac{(t \cos t) \cdot (t \cos t + \sin t) - t \sin t (-t \sin t + \cos t)}{(t \cos t)^3}$$

$$= \frac{t^2 \cos^2 t + t^2 \sin^2 t}{(t \cos t)^3}$$

$$= \frac{t^2 (\cos^2 t + \sin^2 t)}{t^3 \cos^3 t}$$

$$\cos^2 t + \sin^2 t = 1$$

$$\frac{d^2 y}{dx^2} = \frac{t^2}{t^3 \cos t} = \frac{1}{t \cos t}$$

Recall:  $\frac{1}{R} = \frac{d^2 y / dx^2}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}$

$$R = \left[1 + \left(\frac{t \sin t}{t \cos t}\right)^2\right]^{3/2} \cdot \frac{1}{\cos t}$$

$$= \left[1 + \frac{t^2 \sin^2 t}{t^2 \cos^2 t}\right]^{3/2} \times \frac{\cos t}{1}$$

$$= \left[1 + \frac{t^2 (\sin^2 t)}{t^2 (\cos^2 t)}\right]^{3/2} \times \cos t$$

$$= \left[1 + \frac{\sin^2 t}{\cos^2 t}\right]^{3/2} \times \cos t$$

$$= \frac{\cos^2 t + \sin^2 t}{\cos^2 t} = \left[1 + \tan^2 t\right]^{3/2} \times \cos t$$

$$= \left[\sqrt{1} + \sqrt{\tan^2 t}\right]^3 \times \cos t$$

$$= (1 + \tan^2 t)^3 \times \cos t$$

$$= (1 + \tan^2 t) \times \cos t$$

$$= 1 \times \cos t + (\tan^2 t) \cos t$$

$$= \cos t + \frac{\sin^2 t}{\cos^2 t} \cos t$$

$$= \cos t + \frac{\sin^2 t}{\cos t}$$

$$\frac{\cos^2 t + \sin^2 t}{\cos t}$$

$$R = \frac{\cos^2 t + \sin^2 t}{\cos t}$$

$$\text{ii) } \tan \theta = \frac{dy}{dx} = \frac{k \sin t}{k \cos t} = \tan t$$

$$1 \tan \theta = \tan t$$

$$\theta = t$$

$$x_1 = \cos t + k \sin t$$

$$y_1 = \sin t - k \cos t$$

$$h = \cos t + k \sin t - k \sin t$$

$$h = \cos t$$

$$k = \sin t - k \cos t + k \cos t$$

$$= \sin t$$