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16/ENG04/031
Elect. Elect Engr.
ENG 281 Assignment.

Question:

1.) The parametric equations of a curve are as given in equations 1 and 2

$$x = \cos t + t\sin t$$
$$y = \sin t - t\cos t$$

In terms of t, determine:

i.) An expression for the radius of curvature (R), and

ii.) Expressions for the coordinates (h, k) of the centre of curvature.

Solution

$$x = \cos t + t\sin t$$
$$y = \sin t - t\cos t$$

$$\frac{dx}{dt} = -\sin t + (t\cos t + \sin t(1)).$$

$$\frac{dx}{dt} = -\sin t + t\cos t + \sin t$$

$$\frac{dx}{dt} = t\cos t$$

$$\Rightarrow \frac{dy}{dt} = \cos t - (-t\sin t + \cos t(1))$$

$$\therefore \frac{dy}{dt} = t\sin t.$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

$$= \frac{t\sin t}{t\cos t} = \frac{\sin t}{\cos t}$$

$$\Rightarrow \cancel{\frac{d^2y}{dx^2}}\ \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{\sin t}{\cos t}\right)\cdot\frac{dt}{dt}$$

$$= \frac{V\frac{dy}{dx} - U\frac{dv}{dx}}{V^2}$$

$$= \frac{V\frac{dy}{dt} - U\frac{dv}{dt}}{V^2}$$

where, $U = \sin t$, $V_1 = \cos t$

$$\Rightarrow \frac{dy}{dt} = \cos t, \quad \frac{dv}{dt} = -\sin t$$

$$\frac{d^2y}{dx^2} = \frac{\cos t(\cos t) - \sin t(-\sin t)}{\cos^2 t}$$

$$\frac{d^2y}{dx^2} = \frac{\cos^2 t + \sin^2 t}{\cos^2 t}$$

Recall: $\cos^2\theta + \sin^2\theta = 1$, $\Rightarrow \cos^2 t + \sin^2 t = 1$,

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{\cos^2 t} \times \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{1}{\cos^2 t} \times \frac{1}{t\cos t}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{1}{t\cos^2 t}.$$

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i.) Recall that, $R = \dfrac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\dfrac{d^2y}{dx^2}}$

$$R = \left[1 + \left(\frac{\sin t}{\cos t}\right)^2\right]^{3/2} \div \frac{d^2y}{dx^2}$$

$$\Rightarrow R = \left[1 + \frac{\sin^2 t}{\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$\Rightarrow R = \left[\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$\Rightarrow R = \left(\frac{1}{\cos^2 t}\right)^{3/2} \times t\cos^3 t$$

$$\Rightarrow R = \frac{1}{(\cos t)^{2\times 3/2}} \times t\cos^3 t$$

$$\Rightarrow R = \frac{1}{\cancel{\cos^3 t}} \times t\,\cancel{\cos^3 t}$$

$$\Rightarrow R = 1 \times t$$

$$\therefore R = \underline{\underline{t}}.$$

Therefore, the radius of curvature (R) is $\underline{\underline{t}}$.

ii) Recall that, $h = x - R\sin\theta$

$K = y + R\cos\theta$

where $R = t$

$$\theta = \tan^{-1}\left(\frac{dy}{dx}\right)$$

$$= \tan^{-1}(\tan t)$$

$$= t$$

$$\Rightarrow h = \cancel{\cos t + \sin t}\ \cos t + \sin t - t\sin t$$

$$= \cos t$$

$$\Rightarrow K = \sin t - t\cos t + t\cos t$$

$$= \sin t$$

Therefore, the coordinates $(h, K)$ of the centre of curvature ~~is~~ are $(\cos t, \sin t)$.

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