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16/ENG04/050
Elect/Elect Engineering

1a $\lim_{x \to \frac{\pi}{2}} \left[\frac{\left(x^2 - \frac{\pi}{4}\right) \sin(\cos x)}{x - \frac{\pi}{2}} \right]$

$= \left[\frac{\left(\left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4}\right) \sin\left(\cos\frac{\pi}{2}\right)}{\frac{\pi}{2} - \frac{\pi}{2}} \right]$

$= \left[\frac{\left(\left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4}\right) \sin\left(\cos\frac{\pi}{2}\right)}{\frac{\pi}{2} - \frac{\pi}{2}} \right] = \frac{\left(\frac{\pi^2}{4} - \frac{\pi}{4}\right) \sin\left(\cos\frac{\pi}{2}\right)}{0}$

Using L'Hopital's rule $u\frac{dv}{dx} + v\frac{du}{dx}$

$\frac{dy}{dx}$ Let $u = x^2 - \frac{\pi}{4}$ and $v = \sin(\cos x)$

$\frac{du}{dx} = 2x$, $\frac{dy}{dx} = ?$

$\frac{d}{dx} = \sin(\cos x)$. Let $\cos x = w$

$v = \sin w$

$\frac{dv}{dw} = \cos w$, $\frac{dw}{dx} = -\sin x$

$\frac{dv}{dx} = \frac{dv}{dw} \times \frac{dw}{dx} = -\sin x \cos(\cos x)$

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$\frac{}{du} \qquad dx$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = -\sin x \cos(\cos x)$$

$$\frac{\left(x^2 - \frac{\pi}{4}\right) x - \sin x \cos(\cos x) + \sin(\cos x)(2x)}{1}$$

$$= \left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4} \times - \sin 90 \cos(\cos 90) + \sin \cos 90 \times 2\left(\frac{\pi}{2}\right)$$

$$= \left(\frac{\pi^2}{4} - \frac{\pi}{4}\right) x - 1 + 0 \times \pi$$

$= -\frac{\pi^2}{4} + \frac{\pi}{4} = \frac{\pi}{4} - \frac{\pi^2}{4}$

$$\lim_{x \to \frac{\pi}{2}} \left[\frac{(x^2 - \frac{\pi}{4})\sin(\cos x)}{x - \pi/2} \right] = \frac{\pi(1-\pi)}{4}$$

1b $\lim_{x \to \pi/2} \ln \left[\frac{\exp(3x^2+2x-1)}{x+1} \right]$

$= \lim_{x \to \pi/2} \ln \exp \left[\frac{(3x-1)(x+1}{x+1^2} \right]$

$= \lim_{x \to \pi/2} \ln [\exp(3x-1)]$

$= \lim_{x \to \frac{\pi}{2}} (3x-1) = 3\left(\frac{\pi}{2}\right) - 1$

$= \frac{3}{2}\pi - 1 = \frac{3\pi - 1}{2}$

$\therefore \lim \ln \left[\exp \frac{(3x^2+2x-1)}{x+1} \right] = \frac{3\pi - 2}{2}$

1c $\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right]$

$= \lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3}}\right]$

$= \cos\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$

$= \cos[\sin^{-1}(0.8660)]$

$= \cos 60^\circ$

$= \frac{1}{2}$

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1d $\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right]$

$\lim_{x \to 4} \left[\frac{(x-4)(x-4)}{(x-4)(x-1)} \right]$

$\lim_{x \to 4} \left[\frac{x-4}{x-1} \right]$

$= \frac{4-4}{4-1}$

$= \frac{0}{3} = 0$

2 Determine whether each of the following series is conv

a $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \dots$

$U_n = \frac{2}{(n+1)(n+2)}$ $\quad U_{n+1} = \frac{2}{(n+2)(n+3)}$

$\frac{U_{n+1}}{U_n} = \frac{2}{(n+2)(n+3)} \times \frac{(n+1)(n+2)}{2}$

$= \frac{n+1}{n+3}$

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \lim_{n\to\infty} \frac{n+1}{n+3}$

$= \frac{\frac{n}{n} + \frac{1}{n}}{\frac{n}{n} + \frac{3}{n}} = \frac{1 + \frac{1}{n}}{1 + \frac{3}{n}} = \frac{1+0}{1+0} = \frac{1}{1} = 1$

Since $\lim_{n\to\infty} \frac{U_{n+1}}{U_{n+1}}$

The series is inconclusive

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2b $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \dots$

$U_n = \frac{2}{n^2}$ $U_{n+1} = \frac{2}{(n+1)^2}$

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{2}{(n+1)^2} \times \frac{n^2}{2}$

$= \frac{n^2}{(n+1)^2} = \frac{n^2}{n^2+2n+1}$

$\lim_{n\to\infty} = \frac{\frac{n^2}{n^2}}{\frac{n^2}{n^2}+\frac{2n}{n^2}+\frac{1}{n^2}} = \frac{1}{1+\frac{2}{n}+\frac{1}{n^2}} = \frac{1}{1} = 1$

$\therefore$ ~~The Series is con~~

From test 1

$$\lim_{n\to\infty} U_n = \lim_{n\to\infty} \frac{2}{n^2}$$

$$\lim_{n\to\infty} \frac{\frac{2}{n^2}}{\frac{n^2}{n^2}} = \frac{0}{1} = 0$$

∴ The series is convergent

2c) $U_n = \frac{1+2n^2}{1+n^2}$

$$\lim_{n\to\infty} \frac{1+2n^2}{1+n^2} = \frac{\frac{1}{n^2}+\frac{2n^2}{n^2}}{\frac{1}{n^2}+\frac{n^2}{n^2}}$$

$$= \frac{0+2}{1} = 2$$

∴ The series is divergent

3 Find the range of values of x for which the series below is absolutely convergent

$$\frac{x}{127} + \frac{x^2}{125} + \dots + \frac{x^n}{(2n+1)^3}$$

$$U_n = \frac{x^n}{(2n+1)^3}, \quad U_{n+1} = \frac{x^{n+1}}{(2n+2)^3}$$

$$\lim \frac{U_{n+1}}{U_n} = \frac{x^{n+1}}{(2n+2)^3} \times \frac{(2n+1)^3}{x^n}$$

$$\frac{x(2n+1)^3}{(2n+2)^3} = \frac{8n^3+12n^2+n+1}{8n^3+24n^2+24n+8}$$

Dividing by n^3

$$= \frac{(8 + {}^{3}/_{n} + {}^{1}/_{n^2} + {}^{1}/_{n^3})}{(8 + {}^{12}/_{n} + {}^{24}/_{n^2} + {}^{8}/_{n^3})}$$

as $n \to \infty$, $1/n \to 0$

$$\frac{8x}{8} \geq x - 1, \quad x < 1$$

1. Evaluate using L'Hopital's Rule

$$\lim_{x \to 0} \left(\frac{\sin x - \cos x}{x^3} \right)$$

$$\lim_{x \to 0} \left(\frac{\sin x - \cos x}{x^3} \right) = \lim_{x \to 0} \left(\frac{\cos x + \sin x}{3x^2} \right)$$

$$= \lim_{x \to 0} \left(\frac{-\sin x + \cos x}{6x} \right)$$

$$= \lim_{x \to 0} \left(\frac{-\cos x - \sin x}{6} \right)$$

$$= \frac{-\cos 0 - \sin 0}{6}$$

$$= \frac{-1 - 0}{6}$$

$$= \frac{-1}{6}$$