

Chima - Boms - Chimbuenyim
 Mechanical Engineering
 16/ENG06/020
 ENG 281

The parametric equations of a curve is given in eqn (1) and (2)

$$x = \cos t + t \sin t$$

$$y = \sin t - t \cos t$$

In terms of t , determine

i) An expression for the radius of curvature (R), and

ii) Expressions for the coordinates (h, k) of the centre of curvature

$$1) \quad R = \left[1 + \left(\frac{dy/dx}{dx/dt} \right)^2 \right]^{3/2}$$

$$\frac{dy}{dx} = ? \quad \frac{d^2y}{dx^2} = ?$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$y = \sin t - t \cos t$$

$$\frac{dy}{dt} = \cos t - [-t \sin t + \cos t]$$

$$= \cos t + t \sin t - \cos t$$

$$\frac{dy}{dt} = t \sin t$$

$$x = \cos t + t \sin t$$

$$\frac{dx}{dt} = -\sin t + [\cos t + \sin t]$$

$$= -\sin t + \cos t + \sin t$$

$$\frac{dx}{dt} = \cos t$$

$$\frac{dy}{dx} = \frac{t \sin t}{\cos t} + \cos t$$

$$\frac{dy}{dx} = \frac{t \sin t}{\cos t} + \cos t$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dt} \left(\frac{dy}{dx} \right) \cdot \frac{dt}{dx}$$

$$= \frac{d}{dt} \left(\frac{t \sin t}{\cos t} + \cos t \right) \times \frac{dt}{dx}$$

$$= \frac{(\cos t)(\cos t) - \sin t(-\sin t)}{(\cos t)^2} \times \frac{1}{\cos t}$$

$$= \frac{\cos^2 + \sin^2}{\cos^2 t} \times \frac{1}{t \cos t}$$

$$\frac{d^2 y}{dx^2} = \frac{1}{t \cos^2 t}$$

$$R = \frac{[1 + \left(\frac{dy}{dx}\right)^2]^{3/2}}{d^2 y / dx^2}$$

$$= [1 + \left(\frac{dy}{dx}\right)^2]^{3/2} \div \frac{dy}{dx^2}$$

$$= \left[\frac{1}{1} + \left(\frac{\sin t}{\cos t}\right)^2 \right]^{3/2} \div \frac{1}{t \cos^3 t}$$

$$= \left[\frac{1}{1} + \frac{\sin^2 t}{\cos^2 t} \right]^{3/2} \times t \cos^3 t$$

$$= \left[\frac{\cos^2 t + \sin^2 t}{\cos^2 t} \right]^{3/2} \times t \cos^3 t$$

$$= \left(\frac{1}{\cos^2 t} \right)^{3/2} \times t \cos^3 t$$

$$= \frac{1}{(\cos^2 t)^{3/2}} \times t \cos^3 t$$

$$= \frac{t \cos^3 t}{\cos^3 t}$$

$$R = t$$

ii) Centre of Curvature

$$h = x - R \sin \theta \quad \text{--- (1)}$$

$$k = y + R \cos \theta \quad \text{--- (2)}$$

$$x = \cos t + t \sin t$$

$$y = \sin t - t \cos t$$

$$R = t, \quad \theta = t$$

Sub x, R, θ into eqn (1) and y, R, θ into eqn (2)

$$h = (\cos t + t \sin t) - t \sin t$$

$$h = \cos t$$

$$k = (\sin t - t \cos t) + t \cos t$$

$$k = \sin t$$

$$(h, k) = (\cos t, \sin t)$$