Recall θ is also t ; $R = t$.

(h, k)

$h = x - R\sin\theta$

$k = y + R\cos\theta$

$x = \cos t + t\sin t$

$y = \sin t - t\cos t$

$h = \cos t + t\sin t - (t\sin t)$

$k = \sin t - t\cos t + (t\cos t)$

$h = \cos t + t\sin t - t\sin t = \cos t$

$k = \sin t - t\cos t + t\cos t = \sin t$

$(h, k) = (\cos t, \sin t)$.

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1 Parametric Equation of a curve as given as eqn 1 & 2.

$x = \cos t + \sin t$ ... eqn(i)

$y = \sin t - t\cos t$ ... eqn(ii)

Determine R.

Sol

$x = \cos t + \sin t$.

$\frac{\partial x}{\partial t} = \frac{\partial u}{\partial t} + \frac{\partial v}{\partial t}$ $= -\sin t$

$\frac{\partial v}{\partial t}$ — using product rule $t = p$ $\sin t = q$.

$\frac{\partial v}{\partial t} = p\frac{dq}{dt} + q\frac{\partial p}{\partial t}$ $= t\cos t + \sin t \cdot 1$

$= t\cos t + \sin t$.

$\frac{\partial v}{dt} = \sin t + t\cos t$; $\frac{\partial x}{\partial t} = \sin t + t\cos t$.

$\frac{dx}{dt} = -\sin t + \sin t + t\cos t$

$\frac{dx}{dt} = t\cos t$

$y = \sin t - t\cos t$.

$\frac{\partial y}{\partial t} = \frac{\partial u}{\partial t} - \frac{\partial v}{\partial t}$

$v = t\cos t$, i.e $q = \cos t$.

$\frac{dq}{dt} = -\sin t$; $\frac{dp}{\partial t} = 1$.

$\frac{dv}{dt} = p\frac{dq}{dt} + q\frac{\partial p}{\partial t}$ $= t \cdot -\sin t + \cos t \cdot 1$

$= -t\sin t + \cos t$.

$dy/dt = \cos t. - (t\sin t + \cos t) = t\sin t.$

$dy/dt = t\sin t.$

$dy/dx = dy/dt + dt/dx.$

but $\dfrac{dt}{dx} = \left(\dfrac{dx}{dt}\right)^{-1},$

$\dfrac{dy}{dx} = t\sin t \times (t\cos t)^{-1}. \quad \dfrac{dy}{dx} = \dfrac{t\sin t}{t\cos t} = \tan t \quad \dfrac{\partial y}{\partial x} = \tan t$

$= \tan\theta$

$\therefore \theta = \tan^{-1}\tan t$

$= t.$

$\dfrac{d^2y}{dx^2} = \dfrac{d(\partial y/\partial x)}{dt} \div \dfrac{\partial x}{\partial t},$

$-\sec^2 t \div t\cos t$

$= \dfrac{\sec^2 t}{t\cos t} = \dfrac{1}{t\cos^3 t}.$

$R = \dfrac{\left[1 + (\partial y/\partial x)^2\right]^{3/2}}{d^2y/\partial x^2} = \dfrac{\left[1 + (\sin^2 t/\cos t)^2\right]^{3/2}}{\dfrac{1}{t\cos^3 t}}$

$= \dfrac{(1 + \sin^2 t/\cos^2 t)^{3/2}}{1/t\cos^3 t} = \left(\dfrac{\cos^2 t/\cos^2 t + \sin^2 t/\cos^2 t}{1/t\cos^3 t}\right)^{3/2}$

$= \left[\dfrac{\sin^2 t + \cos^2 t/\cos^2 t}{1/t\cos^3 t}\right]^{3/2} = \left[\dfrac{1/\cos^2 t}{1/t\cos^3 t}\right]^{3/2}.$

$= \left[\dfrac{1}{(\cos t)^2}\right]^{3/2} \div 1/t(\cos t)^3,$

$= \dfrac{1}{\cancel{\cos t}^3} + \dfrac{\cancel{\cos t}^3}{1} = t$

$R = t$