15/ENG04/025
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ELECTRICAL/ELECTRONICS ENGINEERING

(1) $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

$m^2 + 4m + 5 = 0$.

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-4 \pm \sqrt{16 - 20}}{2}$

$m = \frac{-4 \pm \sqrt{-4}}{2}$

$m = \frac{-4 \pm 2j}{2}$

$m_1 = -2 \pm j$

CF $y = e^{-2\theta}(A\cos\theta + B\sin\theta)$

PI $y = C\sin\theta + D\cos\theta$.

$\frac{dy}{dx} = C\cos\theta - D\sin\theta$.

$\frac{d^2y}{dx^2} = -C\sin\theta - D\cos\theta$.

$\Rightarrow -C\sin\theta - D\cos\theta + 4(C\cos\theta - D\sin\theta) + 5(C\sin\theta + D\cos\theta) = 6\sin\theta$.

$\Rightarrow -C\sin\theta - D\cos\theta + 4C\cos\theta - 4D\sin\theta + 5C\sin\theta + 5D\cos\theta = 6\sin\theta$

comparing coefficient

$-C - 4D + 5C = 6$ — (i)

$-D + 4C + 5D = 0$ — (ii)

using elimination method

$4C - 4D = 6$ — (iv)

$-4C + 4D = 0$ — (v)

$0 - 8D = 6$

$D = \frac{-6}{8} = \frac{-3}{4}$

from equation (v)

$4C + 4D = 0$

$C = \frac{-4D}{4}$

$C = \frac{+4 \times +3}{4} \times \frac{1}{4}$

$C = \frac{3}{4}$

P.I $y = \frac{3}{4}\sin\theta - \frac{3}{4}\cos\theta$

$= \frac{3}{4}(\sin\theta - \cos\theta)$

G.S = C.F + P.I

G.S $y = e^{-2\theta}(A\cos\theta + B\sin\theta) + \frac{3}{4}(\sin\theta - \cos\theta)$

(ii) P.I $y = \frac{3}{4}(\sin\theta - \cos\theta)$ for $\theta = 0$ to $270°$

θ	y
0	-0.7500
60	0.4857
120	-0.1752
195	-0.5671
270	-0.8703

(iii) at steady state $\frac{dy}{d\theta} = 0 \quad \theta = \alpha$.

$$y = e^{-2\theta}(A\cos\theta + B\sin\theta) + \frac{3}{4}(\sin\theta - \cos\theta).$$

$$\frac{dy}{d\theta} = e^{-2\theta}(-A\sin\theta + B\cos\theta) + (A\cos\theta + B\sin\theta).-2e^{-2\theta} + \frac{3}{4} + \frac{3}{4}(\cos\theta + \sin\theta)$$

at $\frac{dy}{d\theta} = 0$, $\theta = \alpha$

$$0 = \frac{3}{4}(\cos\theta + \sin\theta).$$

$$0 = \cos\theta + \sin\theta$$

$$\sin\theta = -\cos\theta$$

$$-\frac{\sin\theta}{\cos\theta} = 1$$

$$-\tan\theta = 1$$

$$\tan\theta = -1$$

$$\theta = \tan^{-1}(-1)$$

$$\theta = -45^\circ$$

$\therefore \theta = 135^\circ$ or 315°.

(2) $EI\frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2$

$$EI\,m^2 = 0$$

$$m^2 = 0$$

$$m = 0$$

CF $y = e^{0x}(A + Bx)$.

$$y = A + Bx$$

PI $y = Mx^2 + Nx^3 + Ox^4$

$$\frac{dy}{dx} = 2Mx + 3Nx^2 + 4Ox^3$$

$$\frac{d^2y}{dx^2} = 2M + 6Nx + 12Ox^2$$

$$EI\,(2M + 6Nx + 120x^2) = \frac{w}{2}(L-x)^2$$

$$2MEI + 6NxEI + 120x^2EI = \frac{w}{2}(L^2 - 2Lx + x^2)$$

cross multiply

$$4MEI + 12NxEI + 240x^2EI = w(L^2 - 2Lx + x^2)$$

comparing coefficients

$$240EI = w$$

$$0 = \frac{w}{24EI}$$

$$12NEI = -2wL$$

$$N = \frac{-2wL}{12EI}$$

$$N = \frac{-wL}{6EI}$$

$$4MEI = wL^2$$

$$M = \frac{wL^2}{4EI}$$

$$y = \left(\frac{wL^2}{4EI}\right)x^2 + \left(\frac{-wL}{6EI}\right)x^3 + \left(\frac{w}{24EI}\right)x^4$$

$$y = \frac{wL^2x^2}{4EI} - \frac{wLx^3}{6EI} + \frac{wx^4}{24EI}$$

$$y = \frac{6wL^2x^2 - 4wLx^3 + wx^4}{24EI}$$

$$y = \frac{w}{24EI}(6L^2x^2 - 4Lx^3 + x^4)$$

G.S $\quad y = A + Bx + \frac{w}{24EI}(6L^2x^2 - 4Lx^3 + x^4)$

at $y=0$, $x=0$, $\frac{dy}{dx} = 0$

$$0 = A + B(0) + \frac{w}{24EI}\left(6L^2(0)^2 - 4L(0)^3 + 0^4\right)$$

$$A = 0$$

$$\frac{dy}{dx} = B + \frac{w}{24EI}\left(12L^2x - 12Lx^2 + 4x^3\right)$$

$$0 = B + \frac{w}{24EI}\left(12L^2(0) - 12L(0)^2 + 4(0)\right)$$

$$B = 0$$

Particular solution

$$y = \frac{w}{24EI}\left(6L^2x^2 - 4Lx^3 + x^4\right)$$

$$y = \frac{w}{24EI}x^2\left(6L^2 - 4Lx + x^2\right)$$

when $x = L$.

$$y = \frac{wL^2}{24EI}\left(6L^2 - 4L^2 + L^2\right)$$

$$y = \frac{wL^2}{24EI}\left(3L^2\right)$$

$$y = \frac{3wL^4}{24EI}$$

$$y = \frac{wL^4}{8EI}$$