

$$m = \pm 2$$

$$y = A \cosh 2x + B \sinh 2x$$

$$\text{P.I. } y = Ce^{3x}$$

$$\frac{dy}{dx} = 3Ce^{3x}$$

$$\frac{d^2y}{dx^2} = 9Ce^{3x} = 9Ce^{3x} + 0(3Ce^{3x}) - 4(Ce^{3x}) + 0e^{3x}$$

$$\frac{d^2y}{dx^2} = 9Ce^{3x} - 4Ce^{3x} = 5Ce^{3x}$$

$$5Ce^{3x} = 10e^{3x}, \quad C = \frac{10e^{3x}}{5e^{3x}}$$

$$C = 2, \quad y = 2e^{3x}$$

$$e^{-2x} = C.F. + P.I.$$

$$y = A \cosh 2x + B \sinh 2x + 2e^{3x}$$

$$3 \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + y = e^{-2x}$$

$$\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = 0$$

$$m^2 + 2m + 1 = 0, \quad m^2 + m + m + 1 = 0$$

$$m(m+1) + 1(m+1) = 0, \quad m = -1 \text{ (twice)}$$

$$\text{P.I. } y = e^{-2x} (A + Bx)$$

$$e^{2x} y = Ce^{-2x}$$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4Ce^{-2x}$$

$$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$$

$$C = 1$$

$$y = e^{-2x}$$

$$G.S = C.F + P.I$$

$$y = e^{-x}(A + Bx) + e^{-2x}$$

$$4) \frac{d^2y}{dx^2} + 25y = 5x^2 + 2x$$

$$\frac{d^2y}{dx^2} + 25y = 0$$

$$m^2 + 25 = 0$$

$$m^2 + 0 + 25 = 0, \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{0 \pm \sqrt{0 - 4(25)}}{2(1)}, \frac{-0 \pm \sqrt{-25 \times 4}}{2}$$

$$= \frac{0 \pm \sqrt{100}}{2}, \frac{0 \pm \sqrt{100}}{2}, m = \pm \sqrt{25} \quad m = \pm 5$$

$$C.F \quad y = A \cos 5x + B \sin 5x$$

$$P.I = y = cx^2 + dx + e, \frac{dy}{dx} = 2cx + d$$

$$\frac{d^2y}{dx^2} = 2c$$

$$2c + 0 (2cm + d) + 25(cx^2 + dx + e) = 5x^2 + 2x$$

$$2c + 25(cx^2 + dx + e) = 5x^2 + 2x$$

$$2c + 25c \cdot x^2 + 25dx + 25e = 5x^2 + 2x$$

Comparing coefficients.

$$2c + 25e = 0 \quad \text{--- (i)}$$

$$25c = 5 = 5 \quad \text{--- (ii)}$$

$$c = \frac{5}{25}, c = \frac{1}{5} \quad \text{--- (iii)}$$

$$25\Delta = 1 \Rightarrow \Delta = 1/25 \dots \textcircled{iii}$$

Sub equ \textcircled{ii} into \textcircled{i}

$$2\left(\frac{1}{5}\right) + 25\varepsilon = 0$$

$$\frac{2}{5} + 25\varepsilon = 0$$

$$\frac{1}{25} \times 25\varepsilon = -\frac{2}{5} \times \frac{1}{25}$$

$$\varepsilon = \frac{-2}{125}$$

$$P.I = \frac{5}{25} x^2 + \frac{1}{25x} - \frac{2}{125}$$

$$\frac{1}{25} \cdot [25x^2 + 5x - 2]$$

$$\text{G.S. } y = A \cos 5x + B \sin 5x + \frac{1}{25} [25x^2 + 5x - 2]$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4 \sin x$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

$$m^2 - 2m + 1 = 0$$

$$m^2 - m - m + 1 = 0$$

$$m(m-1) - 1(m-1) = 0$$

$$m-1=0, m-1=0$$

$$m=1 \text{ (twice)}$$

$$\text{C.F. } y = e^{mx} [A + Bx]$$

$$\text{P.S. } (C \cos x + D \sin x) = y$$

$$\frac{dy}{dx} = (\sin x + 1) \cos x$$

$$\frac{d^2 y}{dx^2} = -C \cos x - 5 \sin x$$

$$\int [-C \cos x - 5 \sin x] \cdot 2 \int [-C \sin x + \Delta \cos x] + \int [C \cos x + 5 \sin x]$$

$$= 4 \sin x$$

$$-C \cos x - 5 \sin x + 2 \sin x - 2 \Delta \cos x + C \cos x + 5 \sin x$$

$$= 4 \sin x$$

$$\cos x [-C + 2\Delta + C] + \sin x [-5 + 2(-\Delta) + 5] = 4 \sin x$$

$$\cos x [-2\Delta] + \sin x (2\Delta) = -4 \sin x$$

Comparing coefficients

$$-2\Delta = -4$$

$$-\Delta = -2$$

$$2\Delta = 4$$

$$\Delta = 2$$

$$P.F. = 2 \cos x + 0 (\sin x)$$

$$y = 2 \cos x$$

$$\text{Ans} = y = e^{2x} (A + Bx) + 2 \cos x$$

$$e) \frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 5y = 2e^{-2x}$$

given that $x=0, y=1, \frac{dy}{dx} = -2$

$$\frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 5y = 0$$

$$m^2 + 4m + 5 = 0$$

$$m = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}, m = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$$

$$m = \frac{-4 \pm \sqrt{16-20}}{2}$$

$$m = -4 \pm \sqrt{-4}$$

$$m = \frac{-4 \pm j\frac{2}{2}}{2}$$

$$m = -2 \pm j$$

$$y = e^{-2x} (A \cos x + B \sin x)$$

$$P.I.; y = ce^{-2x}$$

$$\frac{dy}{dx} = -2ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4ce^{-2x}$$

$$4ce^{-2x} + 4c - 2(-2ce^{-2x}) + 5(ce^{-2x}) = 2e^{-2x}$$

$$4ce^{-2x} - 8ce^{-2x} + 5ce^{-2x} = 2e^{-2x}$$

$$ce^{-2x}(4-8+5) = 2e^{-2x}$$

$$c = 2$$

$$P.I. = y = 2e^{-2x}$$

$$y = e^{-2x} (A \cos x + B \sin x) + 2e^{-2x}$$

$$P.I. = y = ce^{-2x}$$

$$\frac{dy}{dx} = -2ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4ce^{-2x}$$

$$4ce^{-2x} - 8ce^{-2x} + 5ce^{-2x} = 2e^{-2x}$$

$$ce^{-2x}(4-8+5) = 2e^{-2x}$$

$$ce^{-2x} = 2e^{-2x}$$

$$c = 2$$

$$PI = y = 2e^{-2x}$$

$$y = e^{-2x}(A \cos x + B \sin x) + 2e^{-2x}$$

$$\text{at } x=0, y=1$$

$$1 = e^{-2(0)}(A \cos(0) + B \sin(0)) + 2e^{-0}$$

$$1 = 1(A+0) + 2$$

$$1 = A + 2$$

$$A = -2 + 1$$

$$A = -1$$

$$\therefore \text{at } x=0 \quad \frac{dy}{dx} = -2$$

$$\frac{dy}{dx} = -2e^{-2x}(-A \sin x + B \cos x) - 2e^{-2x}$$

$$-2 = -2e^{-2(0)}(-A \sin(0) + B \cos(0)) - 2e^{-0}$$

$$-2 = -2(0+B) - 2$$

$$-2 = -2B - 2$$

$$-2B = -2 + 2$$

$$B = 0/-2$$

$$B = 0$$

$$y = e^{-2x}(-\cos x + 0 \sin x) + 2e^{-2x}$$

$$y = -e^{-2x} \cos x + 2e^{-2x}$$

$$y = -e^{-2x} \cos x + 2e^{-2x}$$

$$3 \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} - y = 2x - 3$$

$$3 \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} - y = 0$$

$$3m^2 - 2m - 1 = 0$$

$$3m^2 - 3m + m - 1 = 0$$

$$3m(m-1) + 1(m-1) = 0$$

$$(3m+1)(m-1) = 0$$

$$3m+1 = 0$$

$$3m = -1$$

$$m_1 = -1/3, m_2 = 1$$

$$\text{C.F. ; } y = A e^{-1/3x} + B e^{x}$$

$$\text{P.F. ; } y = cx + d$$

$$\frac{dy}{dx} = c, \frac{d^2 y}{dx^2} = 0$$

$$3(0) + (-2c) - (cx + d) = 2x - 3$$

$$0 - 2c - cx - d = 2x - 3$$

$$-2c - d - cx = 2x - 3$$

comparing co-efficients

$$-2c - d = -3 \quad \text{--- (1)}$$

$$-cx = 2x \quad \text{--- (2)}$$

from equ (2)

$$-cx = 2x$$

$$c = -2$$

$$16cx e^{4x} + 4ce^{4x} + 4ce^{4x} - b(4cx e^{4x} + ce^{4x}) + 8c(ce^{4x}) = 8e^{4x}$$

$$16cx e^{4x} + 4ce^{4x} + 4ce^{4x} - 24cx e^{4x} - bce^{4x} + 8cx e^{4x} = 8e^{4x}$$

$$16cx e^{4x} - 24cx e^{4x} + 8cx e^{4x} + 4ce^{4x} + 4ce^{4x} - bce^{4x} = 8e^{4x}$$

$$-8cx e^{4x} - bce^{4x} = 8e^{4x}$$

$$-8c = 8$$

$$c = -1$$

$$c = 8/2$$

$$c = 4$$

$$PI = y = 4xe^{4x}$$

$$C.D = C.F + P.I$$

$$= Ae^{2x} + Be^{4x} + 4xe^{4x}$$

from eqn ①

$$-2e^{-\Delta} = 3$$

$$-2(2) - \Delta = -3$$

$$-4 - \Delta = -3$$

$$-\Delta = -3 + 4$$

$$-\Delta = -7$$

$$C = -2, \Delta = 7$$

$$y = Cx + \Delta$$

$$PI = y = -2x + 7$$

$$C.S = y = Ae^{2x} + Be^{-1/3x} - 2x + 7$$

$$8) \frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$$

$$m^2 - 6m + 8 = 0$$

$$(m^2 - 4m) - (2m + 8) = 0$$

$$m(m - 4) - 2(m + 4) = 0$$

$$(m - 2)(m - 4) = 0$$

$$m_1 = 2, m_2 = 4$$

$$PI = y = Ce^{4x}$$

$$\frac{dy}{dx} = 4Ce^{4x} + Ce^{4x}$$

$$\frac{d^2y}{dx^2} = 16Ce^{4x} + 4Ce^{4x} + 4Ce^{4x}$$

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$$

15/10/2020 P.

15/10/2020

PETROLEUM ENGINEERING

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Q) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$

$$m^2 - m - 2 = 0$$

$$m^2 + m - 2m - 2 = 0$$

$$m(m+1) - 2(m+1) = 0$$

$$m-2=0, m+1=0$$

$$m=2, m=-1, \text{ C.F. } y = Ae^{2x} + Be^{-x}$$

$$\text{P.I. } = y = c$$

$$\frac{dy}{dx} = 0, \frac{d^2y}{dx^2} = 0, 0 - 0 - 2c = 8$$

$$-2c = 8, c = -4, \text{ Ans } = \text{C.F.} + \text{P.I.}$$

$$y = Ae^{2x} + Be^{-x} - 4$$

Q) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$$\frac{d^2y}{dx^2} - 4y = 0$$

$$m^2 - 4 = 0$$

$$\sqrt{m^2} = \pm \sqrt{4}$$