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Assignment II

Course:- ENG381 <Engineering Mathematics>.

1) $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

Note: It is a non-homogenous equation.

:- General solution = complementary function + particular integral

:- Complementary function:

$$\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 0$$

Auxillary equation

$$m^2+4m+5=0$$
$$m^2+4m=-5$$
$$m^2+4m+(2)^2=-5+(2)^2$$
$$(m+2)^2=-1$$
$$m+2=\pm\sqrt{-1}$$
$$m+2=\pm i$$

$:-\ m=-2+i \ \&\ m=-2-i$

$:-\ Cf = C_1e^{(-2+i)\theta} + C_2e^{(-2-i)\theta}$

$$y_n = C_1e^{-2\theta+i\theta} + C_2e^{-2\theta-\theta i}$$
$$y_n = C_1e^{-2\theta}e^{i\theta} + C_2e^{-2\theta}e^{-\theta i}$$
$$y_n = e^{-2\theta}[C_1e^{i\theta} + C_2e^{-\theta i}]$$
$$y_h = e^{-2\theta}[A\cos\theta + B\sin\theta]$$
$$y_p = A\cos\theta + B\sin\theta$$
$$y'_p = -A\sin\theta + B\cos\theta$$
$$y''_p = -A\cos\theta - B\sin\theta$$

$$-A\cos\theta - B\sin\theta + 4[-A\sin\theta + B\cos\theta] + 5A\cos\theta + 5B\sin\theta = 6\sin\theta$$
$$-A\cos\theta - B\sin\theta - 4A\sin\theta + 4B\cos\theta + 5A\cos\theta + 5B\sin\theta = 6\sin\theta$$
$$4A\cos\theta + 4B\sin\theta - 4A\sin\theta + 4B\cos\theta = 6\sin\theta$$
$$(-4A+4B)\sin\theta + (4A+4B)\cos\theta = 6\sin\theta$$
$$-4A+4B=6$$
$$4A+4B=0$$
$$8B=6$$
$$:-\ B=6/8$$
$$B=3/4$$

$4A = -4B$

$A = -B$

$A = -3/4$

$y_p = -3/4 \cos\theta + 3/4 \sin\theta$

$y = y_h + y_p$

$y = e^{-2\theta}[A\cos\theta + B\sin\theta] + 3/4 \sin\theta - 3/4\cos\theta$

:- Steady state equation

$y'p = 0$

$\therefore yp' = 3/4\cos\theta + 3/4\sin\theta = 0$

$= 3/4\cos\theta + 3/4\sin\theta = 0$

$3/4\cos\theta = -3/4\sin\theta$

$\cos\theta = -\sin\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{-\cos\theta}{\cos\theta}$

$:- \tan\theta = -1$

2) $EI \, d^2y/dx^2 = w/2 (L-x)^2$

the auxillary equation becomes

$EI m^2 = 0$

$m^2 = 0$

$m = \pm\sqrt{0}$

$m = \pm 0$

$:- y = e^{0x}[A + Bx]$

$y = A + Bx$

$y_p = y = Fx^2 + Gx^3 + Hx^4$

$dy/dx = 2Fx + 3Gx^2 + 4Hx^3$

$d^2y/dx^2 = 2F + 6Gx + 12Hx^2$

$EI[2F + 6Gx + 12Hx^2] = \frac{w}{2}(L-x)^2$

$2FEI + 6GxEI + 12Hx^2EI = \frac{w}{2}(L-x)^2$

$4FEI + 12GxEI + 24Hx^2EI = w(L-x)^2 =$

$$4FEI + 12EIx + 24HEIx^2 = w(L^2 - 2Lx + x^2)$$

$$24HEI = w$$

$$H = \frac{w}{24EI} \quad \text{---(i)}$$

$$12GEI = -2wL$$

$$G = \frac{-2wL}{12EI} = \frac{-wL}{6EI} \quad \text{---(ii)}$$

$$4FEI = wL^2$$

$$F = \frac{wL^2}{4EI}$$

$$y = \left[\frac{wL^2}{4EI}\right]x^2 - \left[\frac{wL}{6EI}\right]x^3 + \left[\frac{w}{24EI}\right]x^4$$

$$= \frac{wL^2x^2}{4EI} - \frac{wLx^2}{6EI} + \frac{wx^4}{24EI}$$

$$= \frac{6wL^2x^2 - 4wLx^3 + wx^4}{24EI}$$

$$G.E = y = A + Bx + \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

at $y = 0$, $x = 0$, $dy/dx = 0$

$$0 = A + B(0) + \frac{w}{24EI}\left[6L^2(0) - 4L(0) + 0\right]$$

$$A = 0$$

$$dy/dx = B + \frac{w}{24EI}\left[12L^2x - 12Lx^2 + 4x^3\right]$$

$$0 = B + \frac{w}{24EI}\left[12(0) - 12(0) + 4(0)\right]$$

$$B = 0$$

$$y_p = \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

$$y_p = \frac{wx^2}{24EI}\left[6L^2 - 4xL + x^2\right]$$

$$y_p = \frac{wx^2}{24EI}\left[x^2 - 4xL + 6L^2\right]$$

when $x = L$

$$y_p = \frac{wL^2}{24EI}\left[L^2 - 4L^2 + 6L^2\right]$$

$$y_p = \frac{wL^2}{24EI}\left[3L^2\right]$$

$$y = \frac{wL^4}{8EI}$$