ENG 381
Aturamu Adeola Emmanuel
15/ENG04/009
Elect/Elect

1.) $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

$m^2 + 4m + 5 = 0$

$\frac{-4 \pm \sqrt{(4)^2 - 4\times1\times5}}{2\times1} = \frac{4 \pm \sqrt{16-20}}{2} = \frac{-4+\sqrt{4}}{2}$

$= \frac{-4 \pm 2}{2} = -2 \pm j$

$m_1 = -2+j$

$m_2 = -2-j$

CF: $y = e^{-2\theta}(C\cos\theta + D\sin\theta)$

PI: $y = C\cos\theta + D\sin\theta$

$\frac{dy}{d\theta} = -C\sin\theta + D\cos\theta$

$\frac{d^2y}{d\theta} = -C\cos\theta - D\sin\theta$

$-C\cos\theta - D\sin\theta + 4(-C\sin\theta + D\cos\theta) + 5(C\cos\theta + D\sin\theta) = 6\sin\theta$

$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5C\cos\theta + 5D\sin\theta = 6\sin\theta$

$-C\cos\theta + 4D\cos\theta + 5C\cos\theta - D\sin\theta - 4C\sin\theta + 5D\sin\theta = 6\sin\theta$

$\cos\theta(-C + 4D + 5C) + \sin\theta(-D - 4C + 5D) = 6\sin\theta$

$\cos\theta(4C - 4D) - \sin\theta(4C + 4D) = 6\sin\theta$

Comparing Coefficients

$4C + 4D = 0$ — 1

$4C + 4D = 6$ — 2

$4C = 4D$

$C = -D$

Sub value of C in eqn 2

$4(D) + 4D = 6$

$8D = 6$

$D = \frac{6}{8}$

$D = \frac{3}{4}$

$C = -\frac{3}{4}$

$y = -\frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$

GS: $y = e^{-2\theta}((C\cos\theta + D\sin\theta) - \frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$

iii) at $\theta = \alpha$ & $\frac{dy}{d\theta} = 0$

$\frac{dy}{d\theta} = e^{-2\theta}[-C\sin\theta + D\cos\theta] + [C\cos\theta + D\sin\theta] - 2e^{-2\theta} + \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$

at $\theta = \alpha$ & $\frac{dy}{d\theta} = 0$

$0 = -\frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$

$-\frac{3}{4}\sin\theta = -\frac{3}{4}\cos\theta$

$-\sin\theta = -\cos\theta$

divide both sides by $\cos\theta$

$\frac{-\sin\theta}{\cos\theta} = \frac{-\cos\theta}{-\cos\theta}$

$-\tan\theta = 1$

$\tan\theta = -1$

$\theta = \tan^{-1}(-1)$

$\theta = -45^\circ$

2). $EI\frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2$

$EI\,m^2 = 0$

$m^2 = 0$

$m = \pm\sqrt{0}$

$m = \pm 0$

$y = e^{0x}(A+Bx)$

C.F. $y = A+Bx$

P.I: $y = Cx^2 + Dx^3 + Ex^4$

$\frac{dy}{dx} = 2Cx + 3Dx^2 + 4Ex^3$

$\frac{d^2y}{dx^2} = 2C + 6Dx + 12Ex^2$

$EI[2C + 6Dx + 12Ex^2] = \frac{w}{2}(L-x)$

$2CEI + 6DEIx + 12EEx^2 = \frac{2w}{2}(L^2 - 2Lx + x^2)$

multiply both sides by 2

$4CEI + 12DEIx + 24EEx^2 = w(L^2 - 2Lx + x^2)$

$24EEI = -w$

$E = \frac{w}{24EI}$ —— 1

$12DEI = -2wL$

$Q = \frac{-2wL}{12EI} = \frac{-wL}{6EI}$ —— 2

$4CEI = wL^2$

$C = \frac{wL^2}{4EI}$

$y = \left[\frac{wL^2}{4EI}\right]x^2 - \left[\frac{wL}{6EI}\right]x^3 + \left[\frac{w}{24EI}\right]x^4$

$$= \frac{wL^2x^2}{4EI} - \frac{wLx^3}{6EI} + \frac{wx^4}{24EI}$$

$$= \frac{6wL^2x^2 - 4wLx^3 + wx^4}{24EI}$$

$$= \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

$$y = A + Bx + \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

at $y = 0$, $x = 0$, $\frac{dy}{dx} = 0$

$0 = A + B(0) + \frac{w}{24EI}\left[6L^2(0) - 4L(0) + 0\right]$

$A = 0$

$\frac{dy}{dx} = B + \frac{w}{24EI}\left[12L^2x - 12Lx^2 + 4x^3\right]$

$0 = B + \frac{w}{24EI}\left[12(0) - 12(0) + 4(0)\right]$

$B = 0$

Particular solution

$$y = \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

$$y = \frac{wx^2}{24EI}\left[6L^2 - 4Lx + x^2\right]$$

$$y = \frac{wx^2}{24EI}\left[x^2 - 4Lx + 6L^2\right]$$

when $x = L$

$$y = \frac{wL^2}{24EI}\left[L^2 - 4L^2 + 6L^2\right]$$

$$y = \frac{wL^4}{8EI}$$