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ASSIGNMENT 2

$$(1) \frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$$

C.F

$$m^2 + 4m + 5 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = m$$

$$\Rightarrow \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)} = m$$

$$\Rightarrow \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$\Rightarrow \frac{-4 \pm \sqrt{-4}}{2}$$

$$\Rightarrow \frac{-2 \pm 2j}{2}$$

$$m = -2 \pm j$$

$$y = e^{-2\theta} (A \cos\theta + B \sin\theta)$$

P.I

$$y = C \sin\theta + D \cos\theta$$

$$\frac{dy}{d\theta} = C \cos\theta - D \sin\theta$$

$$\frac{d^2y}{d\theta^2} = -C \sin\theta - D \cos\theta$$

$$-C \sin\theta - D \cos\theta + 4(C \cos\theta - D \sin\theta) + 5(C \sin\theta + D \cos\theta) = 6 \sin\theta$$
$$-C \sin\theta - D \cos\theta + 4C \cos\theta - 4D \sin\theta + 5C \sin\theta + 5D \cos\theta = 6 \sin\theta$$

Comparing Coefficients

$$-C - 4D + 5C = 6 \quad \dots (i)$$

$$-D + 4C + 5D = 0 \quad \dots (ii)$$

$$4C - 4D = 6$$

$$-4D + 4C = 0$$

$$0 - 8D = 6$$

$$D = -6/8 = -3/4$$

$$4C = -41$$

$$C = -1$$

$$C = -(-3/4) = 3/4$$

$$\begin{aligned} P \cdot I &= \frac{3}{4} \sin \theta - \frac{3}{4} \cos \theta \\ &= \frac{3}{4} (\sin \theta - \cos \theta) \end{aligned}$$

$$\begin{aligned} G \cdot F &= C \cdot F + P \cdot I \\ &= e^{-2\theta} (A \cos \theta + B \sin \theta) + \frac{3}{4} (\sin \theta - \cos \theta) \end{aligned}$$

$$(ii) \quad y = \frac{3}{4} (\sin \theta - \cos \theta)$$

for $\theta = 0$ to 270°

$$(iii) \quad \text{at steady state } \frac{dy}{d\theta} = 0, \theta = \alpha$$

$$y = e^{-2\theta} (A \cos \theta + B \sin \theta) + \frac{3}{4} (\sin \theta - \cos \theta)$$

$$\frac{dy}{d\theta} = e^{-2\theta} (-A \sin \theta + B \cos \theta) + (A \cos \theta + B \sin \theta) \cdot -2e^{-2\theta} + \frac{3}{4} (\cos \theta + \sin \theta)$$

$$\text{at } \theta = \alpha$$

$$\frac{dy}{d\theta} = \frac{3}{4} (\cos \theta + \sin \theta)$$

$$0 = \frac{3}{4} (\cos \theta + \sin \theta)$$

$$0 = \cos \theta + \sin \theta$$

$$\sin \theta = -\cos \theta$$

Dividing through by $\cos \theta$

$$\frac{\sin \theta}{\cos \theta} = \frac{-\cos \theta}{\cos \theta}$$

$$\tan \theta = -1$$

$$\theta = \tan^{-1}(-1)$$

$$\theta = -45^\circ$$

$$\therefore \theta = 135^\circ \text{ or } 315^\circ //$$

$$(c) EI \frac{d^2y}{dx^2} = \frac{w}{2} (L-x)^2$$

C.F

$$EI m^2 = 0$$

$$m^2 = 0$$

$$m = \sqrt{0}$$

$$m = 0$$

$$y = e^{0x} (A + Bx)$$

$$y = A + Bx$$

P.I

$$y = Gx^2 + Hx^3 + Zx^4$$

$$\frac{dy}{dx} = 2Gx + 3Hx^2 + 4Zx^3$$

$$\frac{d^2y}{dx^2} = 2G + 6Hx + 12Zx^2$$

$$EI (2G + 6Hx + 12Zx^2) = \frac{w}{2} (L-x)^2$$

$$2GEI + 6HxEI + 12Zx^2EI = \frac{w}{2} (L^2 - 2Lx + x^2)$$

Multiply through by 2

$$4GEI + 12HxEI + 24Zx^2EI = w(L^2 - 2Lx + x^2)$$

$$4GEI + 12HxEI + 24Zx^2EI = wL^2 - 2wLx + wx^2$$

Comparing Coefficients

$$4GEI = wL^2$$

$$G = \frac{wL^2}{4EI}$$

$$12HEI = -2wL$$

$$H = \frac{-2wL}{12EI} = \frac{-wL}{6EI}$$

$$24ZEI = w$$

$$Z = \frac{w}{24EI}$$

$$y = \left(\frac{wL^2}{4EI} \right) x^2 + \left(\frac{-wL}{6EI} \right) x^3 + \left(\frac{w}{24EI} \right) x^4$$

$$y = \frac{wL^2 x^2}{4EI} - \frac{wL x^3}{6EI} + \frac{w x^4}{24EI}$$

$$y = \frac{6wL^2 x^2 - 4wL x^3 + w x^4}{24EI}$$

$$y = \frac{w}{24EI} (6L^2 x^2 - 4L x^3 + x^4)$$

G.S

$$y = A + Bx + \frac{w}{24EI} (6L^2 x^2 - 4L x^3 + x^4)$$

$$\text{at } y=0, x=0, \frac{dy}{dx}=0$$

$$0 = A + B(0) + \frac{w}{24EI} (6L^2(0)^2 - 4L(0)^3 + (0)^4)$$

$$0 = A$$

$$A = 0$$

$$\frac{dy}{dx} = B + \frac{w}{24EI} (12L^2 x - 12L x^2 + 4x^3)$$

$$0 = B + \frac{w}{24EI} (12L^2(0) - 12L(0)^2 + 4(0)^3)$$

$$B = 0$$

P.S

$$y = \frac{w}{24EI} (6L^2 x^2 - 4L x^3 + x^4)$$

$$y = \frac{w}{24EI} x^2 (6L^2 - 4Lx + x^2)$$

$$x = L$$

$$y = \frac{wL^2}{24EI} (6L^2 - 4L^2 + L^2)$$

$$y = \frac{wL^2}{24EI} (3L^2)$$

$$y = \frac{3wL^4}{24EI}$$

$$y = \frac{wL^4}{8EI} //$$