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1) $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

C.F

$m^2 + 4m + 5 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = m$

$\frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)} = m$

$\frac{-4 \pm \sqrt{16 - 20}}{2} = m$

$\frac{-4}{2} \pm \frac{\sqrt{-4}}{2} = m$

$-2 \pm \frac{j2}{2} = m$

$m = -2 \pm j$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta)$

P.I

$y = C\sin\theta + D\cos\theta$

$\frac{dy}{d\theta} = C\cos\theta - D\sin\theta$

$\frac{d^2y}{d\theta^2} = -C\sin\theta - D\cos\theta$

$-C\sin\theta - D\cos\theta + 4(C\cos\theta - D\sin\theta) + 5(C\sin\theta + D\cos\theta) = 6\sin\theta$

$-C\sin\theta - D\cos\theta + 4C\cos\theta - 4D\sin\theta + 5C\sin\theta + 5D\cos\theta = 6\sin\theta$

Comparing coefficient

$-C + 4D + 5C = 6$ ——— (1)

$-D + 4C + 5D = 0$ ——— (2)

$4C - 4D = 6$ ——— (3)

$4C + 4D = 0$ ——— (4)

$0 - 8D = 6$

$$D = \frac{-6}{8} = -\frac{3}{4}$$

From eq. (4)

$$4C + 4D = 0$$

$$4C = -4D$$

$$C = -D$$

$$C = \frac{3}{4}$$

$$P.I = \frac{3}{4}\sin\theta - \frac{3}{4}\cos\theta \Rightarrow \frac{3}{4}(\sin\theta - \cos\theta)$$

$$G.S = C.F + P.I$$

$$= e^{-2\theta}(A\cos\theta + B\sin\theta) + \frac{3}{4}(\sin\theta - \cos\theta)$$

ii) $y = \frac{3}{4}(\sin\theta - \cos\theta)$

for $\theta = 0$ to $270°$

iii) At steady state $\frac{dy}{d\theta} = 0 \qquad \theta = \alpha$

$$y = e^{-2\theta}(A\cos\theta + B\sin\theta) + \frac{3}{4}(\sin\theta - \cos\theta)$$

$$\frac{dy}{d\theta} = e^{-2\theta}(-A\sin\theta + B\cos\theta) + (A\cos\theta + B\sin\theta).-2e^{-2\theta} + \frac{3}{4}(\cos\theta + \sin\theta)$$

at $\theta = \alpha$

$$\frac{dy}{d\theta} = \frac{3}{4}(\cos\theta + \sin\theta)$$

$$0 = \frac{3}{4}(\cos\theta + \sin\theta)$$

$$0 = \cos\theta + \sin\theta$$

$$\sin\theta = -\cos\theta$$

Dividing all through by $\cos\theta$

$$\frac{\sin\theta}{\cos\theta} = \frac{-\cos\theta}{\cos\theta}$$

$$\tan\theta = -1$$

$$\theta = \tan^{-1} -1$$

$$\theta = -45° \quad \therefore \theta = 135° \text{ or } 315°$$

2) $EI\frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2$

C.F.

$EIm^2 = 0$

$m^2 = 0$

$m = \sqrt{0}$

$m = 0$

$y = e^{0x}(A + Bx)$

$y = A + Bx$

P.I

$y = Gx^2 + Hx^3 + Zx^4$

$\frac{dy}{dx} = 2Gx + 3Hx^2 + 4Zx^3$

$\frac{d^2y}{dx^2} = 2G + 6Hx + 12Zx^2$

$EI(2G + 6Hx + 12Zx^2) = \frac{w}{2}(L-x)^2$

$2GEI + 6HxEI + 12Zx^2EI = \frac{w}{2}(L^2 - 2Lx + x^2)$

$4GEI + 12HxEI + 24Zx^2EI = w(L^2 - 2Lx + x^2)$

$= wL^2 - 2wLx + wx^2$

Comparing coefficient

$24ZEI = w$

$Z = \frac{w}{24EI}$

$12HEI = -2wL$

$H = \frac{-2wL}{12EI} = \frac{-wL}{6EI}$

$4GEI = wL^2$

$G = \frac{wL^2}{4EI}$

$y = \left(\frac{wL^2}{4EI}\right)x^2 + \left(\frac{-wL}{6EI}\right)x^3 + \left(\frac{w}{24EI}\right)x^4$

$y = \frac{wL^2x^2}{4EI} - \frac{wLx^3}{6EI} + \frac{wx^4}{24EI}$

$$y = \frac{6wL^2x^2 - 4wLx^3 + wx^4}{24EI}$$

$$y = \frac{w}{24EI}\left(6L^2x^2 - 4Lx^3 + x^4\right)$$

C.S

$$y = A + Bx + \frac{w}{24EI}\left(6L^2x^2 - 4Lx^3 + x^4\right)$$

at $y = 0,\ x = 0,\ \frac{dy}{dx} = 0$

$$0 = A + B(0) + \frac{w}{24EI}\left(6L^2(0)^2 - 4L(0)^3 + 0^4\right)$$

$$0 = A$$

$$\therefore A = 0$$

$$\frac{dy}{dx} = B + \frac{w}{24EI}\left(12L^2x - 12Lx^2 + 4x^3\right)$$

$$0 = B + \frac{w}{24EI}\left(12L^2(0) - 12L(0)^2 + 4(0)^3\right)$$

$$0 = B$$

$$\therefore B = 0$$

P.S:

$$y = \frac{w}{24EI}\left(6L^2x^2 - 4Lx^3 + x^4\right)$$

$$y = \frac{w}{24EI}x^2\left(6L^2 - 4Lx + x^2\right)$$

at $x = L$

$$y = \frac{wL^2}{24EI}\left(6L^2 - 4L^2 + L^2\right)$$

$$y = \frac{wL^2}{24EI}\left(3L^2\right)$$

$$y = \frac{3wL^4}{24EI}$$

$$y = \frac{wL^4}{8EI}$$