$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

$C = 1 \quad \therefore y = e^{-2x} - P.I$

$\therefore G.S = C.F + P.I$

$= e^{-x}(A + Bx) + e^{-2x}$

Question 4

$\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

C.F

$m^2 + 25 = 0$

$m^2 = -25$

$m = \sqrt{-25}$

$= \sqrt{-1}\sqrt{25}$

$m = j5$

$y = A\cos nx + B\sin nx$

$= A\cos 5x + B\sin 5x$

P.I

$y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

$\Rightarrow 2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

Comparing coefficients

$25C = 5$

$C = \frac{1}{5}$

$25D = 1$

$D = \frac{1}{25}$

$2C + 25E = 0$

$2(\frac{1}{5}) + 25E = 0$

$\frac{d^2y}{dx^2} = 0$

$3(0) - 2(c) - [cx + D] = 2x - 3$

$0 - 2c - cx - D = 2x - 3$

Comparing coefficients

$-c = 2$

$c = -2$

$-2c - D = -3$

$-2(-2) - D = -3$

$4 - D = -3$

$D = 7$

$\Rightarrow y = -2x + 7$

G.S

$= Ae^{-\frac{1}{3}x} + Be^{x} - 2x + 7$

Question 8

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

C.F

$m^2 - 6m + 8 = 0$

$(m-4)(m-2) = 0$

$m_1 = 4 \quad m_2 = 2$

$y = Ae^{4x} + Be^{2x}$

P.I

$y = cxe^{4x}$

$\frac{dy}{dx} = [x \cdot 4e^{4x} + e^{4x}]c$

$\frac{d^2y}{dx^2} = 4c[x \cdot 4e^{4x} + e^{4x}] + 4ce^{4x}$

$= 16cxe^{4x} + 4ce^{4x} + 4ce^{4x}$

$\Rightarrow 16cxe^{4x} + 4ce^{4x} + 4ce^{4x} - 6(4cxe^{4x} + ce^{4x}) + 8cxe^{4x} = 8e^{4x}$

$16cxe^{4x} + 4ce^{4x} + 4ce^{4x} - 24cxe^{4x} - 6ce^{4x} + 8cxe^{4x} = 8e^{4x}$

$16cx + 4c + 4c - 24cx - 6c + 8cx = 8$

$A = 1 - 2$

$A = -1$

$\frac{dy}{dx} = e^{-2x}[-A\sin x + B\cos x] + [A\cos x + B\sin x] \cdot -2e^{-2x} + 2e^{-2x}$

$\frac{dy}{dx} = e^{-2x}(-A\sin x + B\cos x) - 2e^{-2x}(A\cos x + B\sin x) - 4e^{-2x}$

at $x = 0$ $\frac{dy}{dx} = -2$

$-2 = e^{-2(0)}(-A\sin 0 + B\cos 0) - 2e^{-2(0)}(A\cos 0 + B\sin 0) - 4e^{-2(0)}$

$-2 = B - 2A - 4$

$-2 = B - 2(-1) - 4$

$B = -2 - 2 + 4$

$B = 0$

$P.S = e^{-2x}(-\cos x) + 2e^{-2x}$

$= -e^{-2x}\cos x + 2e^{-2x}$

$= e^{-2x}(2 - \cos x)$

Question 7

$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

C.F

$3m^2 - 2m - 1 = 0$

$(3m + 1)(m - 1) = 0$

$3m = -1$ Or $m = 1$

$m_1 = \frac{-1}{3}$ $m_2 = 1$

$y = Ae^{-\frac{1}{3}x} + Be^{x}$

P.F

$y = Cx + D$

$\frac{dy}{dx} = C$

Question 6

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$, given that at $x = 0$ $y = 1$

$\frac{dy}{dx} = -2$

C F

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{16 - 20}}{2}$

$= \frac{-4 \pm \sqrt{-4}}{2}$

$= \frac{-4 \pm \sqrt{-1}\sqrt{4}}{2}$

$= \frac{-4 \pm j2}{2}$

$= -2 \pm j$

$m_1 = -2 + j \quad m_2 = -2 - j$

$y = e^{-2x}(A\cos x + B\sin x)$

P. I

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$4C - 8C + 5C = 2$

$C = 2$

$y = 2e^{-2x}$

G S = C F + P I

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

at $x = 0$ $y = 1$

$25E = -\frac{2}{5}$

$E = -\frac{2}{125}$

$\Rightarrow$ P.I : $y = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

$= \frac{25x^2 + 5x - 2}{125}$

G.S = C.F + P.I

$= A\cos 5x + B\sin 5x + \frac{25x^2 + 5x - 2}{125}$

Question 5

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

C.F

$m^2 - 2m + 1 = 0$

$(m-1)(m-1) = 0$

$m = 1$ twice

$y = e^x(A + Bx)$

P.I

$y = C\cos x + D\sin x$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$\Rightarrow -C\cos x - D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$2C\sin x - 2D\cos x = 4\sin x$

Comparing coefficients

$2C = 4$

$C = 4/2 \Rightarrow C = 2$

$2D = 0$

$D = 0$

P.I $y = 2\cos x + 0\sin x = 2\cos x$

G.S $= e^x(A + Bx) + 2\cos x$

dx

$\frac{d^2y}{dx^2} = 0$

$\Rightarrow 0 - 0 - 2C = 8$

$2C = -8$

$C = \frac{-8}{2}$

$C = -4 \quad \therefore y = -4 \quad - P.I$

$G.S = C.f + P.I$

$= Ae^{2x} + Be^{-x} + (-4)$

$= Ae^{2x} + Be^{-x} - 4$

Question 2

$\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$1 = e^{-(2)}(A\cos 0 + B\sin 0) + 2e^{-1(0)}$

$1 = 1(A) + 2$

$1 = A + 2$

$A = 1 - 2$

$A = -1$

$\frac{dy}{dx} = e^{-2x}[-A\sin x + B\cos x] + [A\cos x + B\sin x] \cdot -2e^{-2x} + 2e^{-2x}$

$\frac{dy}{dx} = e^{-2x}(-A\sin x + B\cos x) - 2e^{-2x}(A\cos x + B\sin x) - 4e^{-2x}$

at $x = 0$ $\frac{dy}{dx} = -2$

$-2 = e^{-2(0)}(-A\sin 0 + B\cos 0) - 2e^{-2(0)}(A\cos 0 + B\sin 0) - 4e^{-2(0)}$

$-2 = B - 2A - 4$

$-2 = B - 2(-1) - 4$

$B = -2 - 2 + 4$

$B = 0$

P.S $= e^{-2x}(-\cos x) + 2e^{-2x}$

$= -e^{-2x}\cos x + 2e^{-2x}$

$= e^{-2x}(2 - \cos x)$

Question 7

$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

C.F

$3m^2 - 2m - 1 = 0$

$(3m + 1)(m - 1) = 0$

$3m = -1$ or $m = 1$

$m_1 = \frac{-1}{3}$ $m_2 = 1$

$y = Ae^{-\frac{1}{3}x} + Be^{x}$

P.F

$y = Cx + D$

$\frac{dy}{dx} = C$

125

G.S = C.F + P.I

$= A\cos 5x + B\sin 5x + \frac{25x^2 + 5x - 2}{125}$

Question 5

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

C.F

$m^2 - 2m + 1 = 0$

$(m-1)(m-1) = 0$

$m = 1$ twice

$y = e^x(A + Bx)$

P.I

$y = C\cos x + D\sin x$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$\Rightarrow -C\cos x - D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$2C\sin x - 2D\cos x = 4\sin x$

Comparing coefficients

$2C = 4$

$C = 4/2 \Rightarrow C = 2$

$2D = 0$

$D = 0$

P.I $y = 2\cos x + 0\sin x = 2\cos x$

G.S $= e^x(A + Bx) + 2\cos x$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

$\Rightarrow 0 - 0 - 2C = 8$

$2C = -8$

$C = \frac{-8}{2}$

$C = -4 \quad \therefore y = -4 \quad - P.I$

$G.S = C.F + P.I$

$= Ae^{2x} + Be^{-x} + (-4)$

$= Ae^{2x} + Be^{-x} - 4$

dx^2

$3(0) - 2(c) - [cx + D] = 2x - 3$

$0 - 2c - cx - D = 2x - 3$

Comparing coefficients

$-c = 2$

$c = -2$

$-2c - D = -3$

$-2(-2) - D = -3$

$4 - D = -3$

$D = 7$

$\Rightarrow y = -2x + 7$

G.S

$= Ae^{-\frac{1}{3}x} + Be^{x} - 2x + 7$

Question 8

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

C.F

$m^2 - 6m + 8 = 0$

$(m-4)(m-2) = 0$

$m_1 = 4 \quad m_2 = 2$

$y = Ae^{4x} + Be^{2x}$

P.I

$y = Cxe^{4x}$

$\frac{dy}{dx} = C[x \cdot 4e^{4x} + e^{4x}]$

$\frac{d^2y}{dx^2} = 4C[x \cdot 4e^{4x} + e^{4x}] + 4Ce^{4x}$

$= 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$\Rightarrow 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 6(4Cxe^{4x} + Ce^{4x}) + 8Cxe^{4x} = 8e^{4x}$

$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$

$16Cx + 4C + 4C - 24Cx - 6C + 8Cx = 8$

$-8Cx + 2C + 8e^{x} = 8$

$2C = 8$

$C = \frac{8}{2}$

$C = 4$

$\therefore y = 4xe^{4x}$

G.S = C.F + P.I

$= Ae^{4x} + Be^{2x} + 4xe^{4x}$

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$, given that at $x = 0$ $y = 1$

$\frac{dy}{dx} = -2$

C F

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{16 - 20}}{2}$

$= \frac{-4 \pm \sqrt{-4}}{2}$

$= \frac{-4 \pm \sqrt{-1}\sqrt{4}}{2}$

$= \frac{-4 \pm j2}{2}$

$= -2 \pm j$

$m_1 = -2 + j \quad m_2 = -2 - j$

$y = e^{-2x}(A\cos x + B\sin x)$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

$C = 1 \quad \therefore y = e^{-2x} - P.I$

$\therefore G.S = C.F + P.I$

$= e^{-x}(A+Bx) + e^{-2x}$

Question 4

$\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

C.F

$m^2 + 25 = 0$

$m^2 = -25$

$m = \sqrt{-25}$

$= \sqrt{-1}\sqrt{25}$

$m = j5$

$y = A\cos 5x + B\sin 5x$

$= A\cos 5x + B\sin 5x$

P.I

$y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

$\Rightarrow 2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

Comparing coefficients

$25C = 5$

$C = \frac{1}{5}$

$25D = 1$

$D = \frac{1}{25}$

$2C + 25E = 0$

$2C(\frac{1}{5}) + 25E = 0$

$m = \sqrt{4}$

$m = \pm 2$

$y = A\cosh 2x + B\sinh 2x$

P.I

$y = ce^{3x}$

$\frac{dy}{dx} = 3ce^{3x}$

$\frac{d^2y}{dx^2} = 9ce^{3x}$

$\Rightarrow 9ce^{3x} - 4(ce^{3x}) = 10e^{3x}$

$9ce^{3x} - 4ce^{3x} = 10e^{3x}$

$9c - 4c = 10$

$5c = 10$

$c = 10/5 = 2 \quad \therefore y = 2e^{3x}$ - P.I

$\therefore$ GF = CF + PI

$= A\cosh 2x + B\sinh 2x + 2e^{3x}$

Question 3

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

C F

$m^2 + 2m + 1 = 0$

$(m+1)(m+1) = 0$

$m = -1$ twice

$y = e^{-x}(A + Bx)$

P.I

$y = ce^{-2x}$

$\frac{dy}{dx} = -2ce^{-2x}$

$\frac{d^2y}{dx^2} = 4ce^{-2x}$