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Matric No: 15121602/032

$$\textcircled{1} \frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$$

Since it is non-homogenous

$$GS = CF + PI$$

Solving first for CF

$$\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 0$$

Auxillary equation

$$m^2 + 4m + 5 = 0$$

$$m^2 + 4m = -5$$

$$m^2 + 4m + 2^2 = -5 + (2)^2$$

$$(m+2)^2 = -1$$

$$m+2 = \pm\sqrt{-1}$$

$$m+2 = \pm i$$

$$m = -2 + i \text{ \& } m = -2 - i$$

$$CF = C_1 e^{(-2+i)\theta} + C_2 e^{(-2-i)\theta}$$

$$y_n = C_1 e^{-2\theta + i\theta} + C_2 e^{-2\theta - i\theta}$$

$$y_n = C_1 e^{-2\theta} \cdot e^{i\theta} + C_2 e^{-2\theta} e^{-i\theta}$$

$$y_n = e^{-2\theta} [C_1 e^{i\theta} + C_2 e^{-i\theta}]$$

$$y_n = e^{-2\theta} [A \cos\theta + B \sin\theta]$$

$$y_p = A \cos\theta + B \sin\theta$$

$$y_p' = -A \sin\theta + B \cos\theta$$

$$y_p'' = -A \cos\theta - B \sin\theta$$

$$-A \cos\theta - B \sin\theta + 4[-A \sin\theta + B \cos\theta] + 5A \cos\theta + 5B \sin\theta = 6 \sin\theta$$

$$4A \cos\theta + 4B \sin\theta - 4A \sin\theta + 4B \cos\theta = 6 \sin\theta$$

$$4A \cos\theta + 4B \sin\theta - 4A \sin\theta + 4B \cos\theta = 6 \sin\theta$$

$$(-4A + 4B) \sin\theta + (4A + 4B) \cos\theta = 6 \sin\theta$$

$$-4A + 4B = 6$$

$$4A + 4B = 0$$

$$8B = 6$$

$$B = \frac{6}{8} = \frac{3}{4}$$

$$4A = -4B$$

$$A = -B$$

$$A = -3/4$$

$$y_p = -3/4 \cos \theta + 3/4 \sin \theta$$

$$y = y_h + y_p$$

$$y = e^{-2\theta} [A \cos \theta + B \sin \theta] + 3/4 \sin \theta - 3/4 \cos \theta$$

Steady state equation

$$y'p = 0$$

$$y_p' = 3/4 \cos \theta + 3/4 \sin \theta = 0$$

$$3/4 \cos \theta + 3/4 \sin \theta = 0$$

$$3/4 \cos \theta = -3/4 \sin \theta$$

$$\cos \theta = -\sin \theta$$

$$\frac{\sin \theta}{\cos \theta} = -\frac{\cos \theta}{\cos \theta}$$

$$\tan \theta = -1$$

$$\textcircled{2} EI \frac{d^2y}{dx^2} = w/2 (L-x)^2$$

The auxiliary equation becomes

$$EI m^2 = 0$$

$$m^2 = 0$$

$$m = \pm \sqrt{0}$$

$$y = e^{0x} [A + Bx]$$

$$y = A + Bx$$

$$y_p = y = Fx^2 + Gx^3 + Hx^4$$

$$\frac{dy}{dx} = 2Fx + 3Gx^2 + 4Hx^3$$

$$\frac{d^2y}{dx^2} = 2F + 6Gx + 12Hx^2$$

$$EI [2F + 6Gx + 12Hx^2] = \frac{w}{2} (L-x)^2$$

$$2FEI + 6GxEI + 12Hx^2EI = \frac{w}{2}(L-x)^2$$

$$4FEI + 12GxEI + 24Hx^2EI = w(L-x)^2$$

$$4FEI + 12EIx + 24HEIx^2 = w(L^2 - 2Lx + x^2)$$

$$24HEI = w$$

$$H = \frac{w}{24EI}$$

$$12GxEI = -2wL$$

$$G = \frac{-2wL}{12EI} = -\frac{wL}{6EI}$$

$$4FEI = wL^2$$

$$F = \frac{wL^2}{4EI}$$

$$y = \left[ \frac{wL^2}{4EI} \right] x^2 - \left[ \frac{wL}{6EI} \right] x^3 + \left[ \frac{w}{24EI} \right] x^4$$

$$= \frac{wL^2x^2}{4EI} - \frac{wLx^3}{6EI} + \frac{wx^4}{24EI}$$

$$\frac{6wL^2x^2 - 4wLx^3 + wx^4}{24EI}$$

$$GE = y = A + Bx + \frac{w}{24EI} [6L^2x^2 - 4Lx^3 + x^4]$$

$$\text{at } y=0, x=0, \frac{dy}{dx} = 0$$

$$0 = A + B(0) + \frac{w}{24EI} [6L^2(0) - 4L(0) + 0]$$

$$A = 0$$

$$\frac{dy}{dx} = B + \frac{w}{24EI} [12L^2x - 12Lx^2 + 4x^3]$$

$$0 = B + \frac{w}{24EI} [12(0) - 12(0) + 4(0)]$$

$$B = 0$$

$$y_p = \frac{w}{24EI} [6L^2x^2 - 4Lx^3 + x^4]$$

$$y_p = \frac{w x^2}{24EI} [x^2 - 4xL + 6L^2]$$

When  $x=L$

$$y_p = \frac{wL^2}{24EI} [L^2 - 4L^2 + 6L^2]$$

$$y_p = \frac{wL^2}{24EI} [3L^2]$$

$$y = \frac{wL^4}{8EI}$$