

EBEGBULEM KENNETH DAVID

16/ENG 06 1022

ENG 281

16/10/2017.

$$x = \cos t + \sin t$$

$$y = \sin t - \cos t$$

$$\frac{dx}{dt} = -\sin t + (\cos t + \sin t)$$

$$\frac{dx}{dt} = -\sin t + \cos t + \sin t$$

$$\frac{dx}{dt} = \cos t$$

$$\frac{dy}{dt} = \cos t + -(-\sin t + \cos t)$$

$$\frac{dy}{dt} = \cos t + \sin t + \cos t$$

$$\frac{dy}{dt} = \sin t + 2\cos t$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{\cancel{t} \sin t}{\cancel{t} \cos t} = \frac{\sin t}{\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dt} \left(\frac{dy}{dx} \right) \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt} \left(\frac{\sin t}{\cos t} \right) \cdot \frac{dt}{dx}$$

$$= \frac{d}{dt} \left(\frac{dy}{dx} \right) \cdot \frac{dt}{dx}$$

$$= \frac{d}{dt} \left(\frac{dy}{dx} \right) \cdot \frac{dt}{dx}$$

where $u = \sin t$, $v = \cos t$.

$$\frac{dy}{dx} = \frac{+v}{u}, \quad \frac{dv}{dt} = -\sin t$$

$$\frac{d^2y}{dx^2} = \frac{\cos t (\cos t) - \sin t (-\sin t)}{\cos^2 t}$$

$$= \frac{\cos^2 t + \sin^2 t}{\cos^2 t}$$

Recall $\cos^2 t + \sin^2 t = 1$

$$\therefore \frac{d^2y}{dx^2} = \frac{1}{\cos^2 t} \times \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{1}{\cos^2 t} \times \frac{1}{\cos t} = \frac{1}{\cos^3 t}$$

$$\text{Recall } R = \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2} \\ \frac{d^2y}{dx^2}$$

$$R = \left[1 + \left(\frac{\sin t}{\cos t} \right)^2 \right]^{3/2} \div \frac{d^2y}{dx^2}$$

$$R = \left[\frac{1 + \sin^2 t}{\cos^2 t} \right]^{3/2} \times \frac{1}{\cos^3 t}$$

$$R = \left[\frac{\cos^2 t + \sin^2 t}{\cos^2 t} \right]^{3/2} \times \frac{1}{\cos^3 t}$$

$$R = \left(\frac{1}{\cos^2 t} \right)^{3/2} \times \frac{1}{\cos^3 t} = \frac{1}{(\cos t)^{2 \times 3/2}} \times \frac{1}{\cos^3 t}$$

$$R = \frac{1}{\cos^3 t} \times \frac{1}{\cos^3 t} = \frac{1}{\cos^6 t} \quad \therefore R = t$$

For the coordinate (h, k) of the centre of curvature, recall $h = x - R \sin \theta$
 $k = y + R \cos \theta$

where $\theta = t$.

$$\theta = \tan^{-1} \left[\frac{dy}{dx} \right] = \tan^{-1} (\tan t) = t$$

$$\therefore h = \cos t - \frac{1}{\cos^3 t} \sin t - \frac{1}{\cos^3 t} \sin t$$

$$h = \cos t$$

$$k = \sin t - \frac{1}{\cos^3 t} \cos t + \frac{1}{\cos^3 t} \cos t$$

$$k = \sin t$$

The co-ordinates for the Centre of Curvature are $(\cos t, \sin t)$

$$\therefore I_{yy} = 0$$

$$\text{Similarly, } I_{xx} = \int \sin^2 t \, dt = \int \frac{1 - \cos 2t}{2} \, dt$$

Moment of Inertia to Simple Area
 Moment of Inertia
 $I = \frac{bd^3}{12}$ $I_{xx} = \frac{bd^3}{12}$