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Mat: 16/ENG02/008
Computer Engineering
ENG 281

1) The parametric equations of a curve are as given in equations 1 and 2

$$x = \cos t + t \sin t$$
$$y = \sin t - t \cos t$$

In terms of t, determine

i) an expression for the radius of curvature.
ii) expressions for co-ordinate (h, k) of the center

Solution//

$$R = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}$$

$$d^2y/dx^2$$

$x = \cos t + t \sin t$, $y = \sin t - t \cos t$

$$dy/dx = dy/dt \div dx/dt$$

$$\frac{dx}{dt} = -\sin t + (t + \cos t + \sin t)$$

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$$= -\sin t + t\cos t + \sin t = t\cos t$$

$$\frac{dy}{dt} = \cos t + (-t - \sin t + \cos t - 1)$$

$$= \cos t + t\sin t - \cos t = t\sin t$$

$$dy/dx = t\sin t \div t\cos t = \frac{\cancel{t}\sin t}{\cancel{t}\cos t} = \frac{\sin t}{\cos t}$$

$$= \tan t$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dt}\left(\frac{dy}{dx}\right) \cdot \frac{dt}{dx}$$

$$= \frac{d}{dt}(\tan t) \times \frac{1}{t\cos t}$$

$$\frac{d^2y}{dx^2} = \sec^2 t \cdot \frac{1}{t\cos t}$$ recall $\sec^2\theta = \frac{1}{\cos^2\theta}$

$$\therefore \frac{d^2y}{dx^2} = \frac{1}{\cos^2 t} \times \frac{1}{t\cos t} = \frac{1}{t\cos^3 t}$$

then $R = \dfrac{\left(1 + (dy/dx)^2\right)^{3/2}}{d^2y/dx^2} = \dfrac{(1+\tan^2 t)^{3/2}}{\frac{1}{t\cos^3 t}}$

recall $1 + \tan^2\theta = \sec^2\theta$

$$\frac{(\sec^2 t)^{3/2}}{(t\cos^3 t)^{-1}} = \frac{\left(\sqrt{\sec^2 t}\right)^3}{(t\cos^3 t)^{-1}}$$

$$= \cancel{x}\ \sec^3 t \times t\cos^3 t$$

where $\sec\theta = \frac{1}{\cos\theta}$

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$$\frac{d^2y}{dx^2} = \frac{1}{\cos^2 t} \times \frac{1}{t\cos t} = \frac{1}{t\cos^3 t}$$

$$R = \frac{\left(1 + (dy/dx)^2\right)^{3/2}}{d^2y/dx^2} = \frac{(1+\tan^2 t)^{3/2}}{\frac{1}{t\cos^3 t}}$$

recall $1 + \tan^2\theta = \sec^2\theta$

$$\frac{(\sec^2 t)^{3/2}}{(t\cos^3 t)^{-1}} = \frac{\left(\sqrt{\sec^2 t}\right)^3}{(t\cos^3 t)^{-1}}$$

$$= \sec^3 t \times t\cos^3 t$$

where $\sec\theta = \frac{1}{\cos\theta}$

$$\cancel{= \sec^2 t = \frac{1}{\cos^2 t}}$$

$$= \frac{1}{\cancel{\cos^3 t}} \times t\cancel{\cos^3 t}$$

$R = t$

1) $h = x_1 - R\sin t$

from $x = \cos t + t\sin t - R\sin t$

$= \cos t + t\sin t - t\sin t$

$= \cos t$

$k = y_1 + R\cos t$

from $y = \sin t - t\cos t$

$k = \sin t - t\cos t = R\cos t$

$k = \sin t - t\cos t + t\cos t$

$k = \sin t$