ENG 381 Assignment 2

(1) $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

(i)

$m^2 + 4m + 5 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2}$

$= \frac{-4 \pm \sqrt{16 - 20}}{2}$

$= \frac{-4 \pm \sqrt{-4}}{2}$

$= \frac{-4 \pm \sqrt{-1}\sqrt{4}}{2}$

$= \frac{-4 \pm 2j}{2}$

$= -2 \pm j$

C.F $y = e^{-2\theta}(C\cos\theta + D\sin\theta)$

P.I. $y = C\cos kx + D\sin kx$

$\cancel{dy/dx = }$

$y = C\cos\theta + D\sin\theta$

$dy/d\theta = -C\sin\theta + D\cos\theta$

$d^2y/d\theta^2 = -C\cos\theta - D\sin\theta$

$(-C\cos\theta - D\sin\theta) + 4(-C\sin\theta + D\cos\theta) + 5(C\cos\theta + D\sin\theta)$
$= 6\sin\theta$

$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5C\cos\theta + 5D\sin\theta = 6\sin\theta$

$(-C + 4D + 5C)\cos\theta + (-D - 4C + 5D)\sin\theta = 6\sin\theta$

$4C + 4D = 0$, $4C = -4D$, $C = -D$

$4D - 4C = 6$

$4D - 4(-D) = 6$

$4D + 4D = 6$

$8D = 6$

$D = 6/8$

$D = 3/4$

$4C + 4D = 0$

$4C + 4(3/4) = 0$

$4C + 3 = 0$

$4C = -3$

$C = -3/4$

P.I $\Rightarrow y = -3/4\cos\theta + 3/4\sin\theta$

General Solution

$y = e^{-2\theta}(C\cos\theta + D\sin\theta) - 3/4\cos\theta + 3/4\sin\theta$

(iii) at $\theta = \infty$

and $\frac{dx}{dt} = 0$ OR $\frac{dy}{d\theta} = 0$

$\frac{dy}{d\theta} = (e^{-2\theta})(-C\sin\theta + D\cos\theta) + (C\cos\theta + D\sin\theta)(-2e^{-2\theta})$
$+ 3/4\sin\theta + 3/4\cos\theta$

$dy/d\theta = (e^{-2\theta})(D\cos\theta - C\sin\theta) - 2e^{-2\theta}(C\cos\theta + D\sin\theta)$
$+ 3/4\sin\theta + 3/4\cos\theta$

at $\theta = \infty$

$0 = 3/4\sin\theta + 3/4\cos\theta$

$-3/4\sin\theta = 3/4\cos\theta$

$-\sin\theta = \cos\theta$

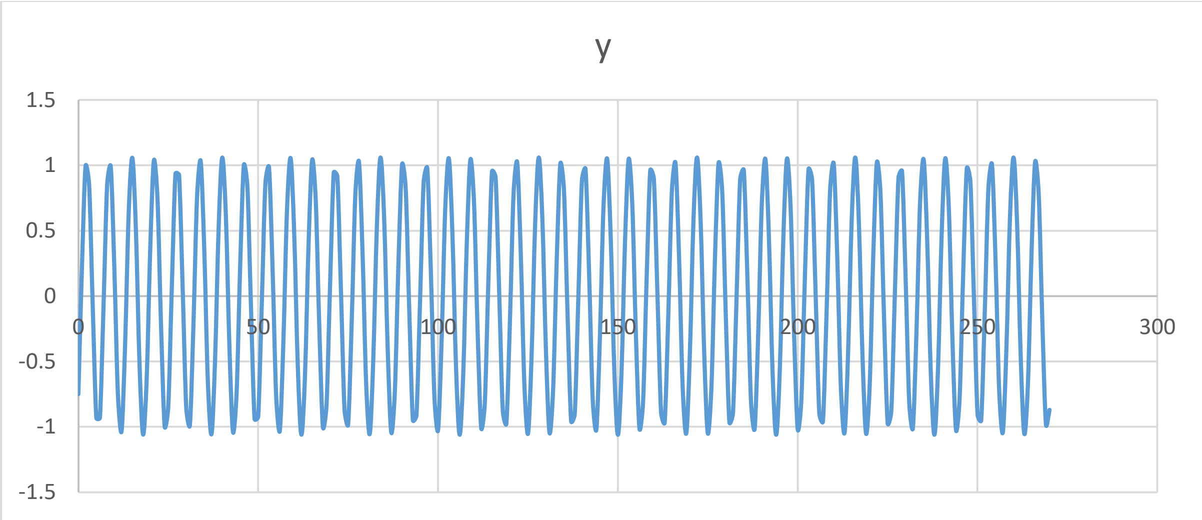
$-\frac{\sin\theta}{\cos\theta} = 1$

$-\tan\theta = 1$

$\tan\theta = -1$

$\theta = \tan^{-1} -1$

$\theta = -45°$


y
1.5
1
0.5
0
-0.5
-1
-1.5
0
50
100
150
200
250
300

$EIm^2 = 0$

$m^2 = 0$

$m = \pm 0$

$y = e^{0x}(A+Bx)$

$y = A+Bx$

P.I

$y = Fx^2 + Gx^3 + Hx^4$

$\frac{dy}{dx} = 2Fx + 3Gx^2 + 4Hx^3$

$\frac{d^2y}{dx^2} = 2F + 6Gx + 12Hx^2$

$EI[2F + 6Gx + 12Hx^2] = \frac{w}{2}(L-x)^2$

$2FEI + 6GEIx + 12HEIx^2 = \frac{w}{2}(L-x^2)$

$2FEI + 6GEIx + 12HEIx^2 = \frac{w}{2}(L^2 - 2Lx + x^2)$

$4FEI + 12GEIx + 24HEIx^2 = w(L^2 - 2Lx + x^2)$

$4FEI + 12GEIx + 24HEIx^2 = wL^2 - 2wLx + wx^2$

$24HEI = w$

$H = \frac{w}{24EI}$

$12GEI = -2wL$

$G = \frac{-2wL}{12EI} = \frac{-wL}{6EI}$

$4FEI = wL^2$

$F = \frac{wL^2}{4EI}$

$y = \left(\frac{wL^2}{4EI}\right)x^2 - \left(\frac{wL}{6EI}\right)x^3 + \left(\frac{w}{24EI}\right)x^4$

$y = \frac{6wL^2x^2 - 4wLx^3 + wx^4}{24EI}$

P.I $\Rightarrow$ $y = \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$

General Solution

$$y = A + Bx + \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

at $y = 0$, and $dy/dx = 0$ at $x = 0$

$0 = A$

$$\frac{dy}{dx} = B + \frac{w}{24EI}\left[12L^2x - 12Lx^2 + 4x^3\right]$$

$0 = B$

Particular solution

$$y = \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

$$y = \frac{wx^2}{24EI}\left[6L^2 - 4Lx + x^2\right]$$

when $x = L$

$$y = \frac{wL^2}{24EI}\left[6L^2 - 4L^2 + L^2\right]$$

$$y = \frac{wL^4}{24EI}\left[3\right]$$

$$\boxed{y = \frac{wL^4}{8EI}}$$