(8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$m^2 - 6m + 8 = 0$

$m^2 - 4m - 2m + 8 = 0$

$m(m-4) - 2(m-4)$

$m = 4 \quad m = 2$

$\therefore y = Ae^{4x} + Be^{2x}$

$y = Ce^{4x}$

$\frac{dy}{dx} = C_x(4e^{4x}) + e^{4x} \cdot C$

$\frac{dy}{dx} = Ce^{4x} + 4Cxe^{4x}$

$\frac{d^2y}{dx^2} = 4Ce^{4x} + [4Cx(4e^{4x}) + 4Ce^{4x}]$

$= 4Ce^{4x} + 16Cxe^{4x} + 4Ce^{4x}$

$= 16Cxe^{4x} + 8Ce^{4x}$

Sub

$16Cxe^{4x} + 8Ce^{4x} - 6(Ce^{4x} + 4Cxe^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$

$16Cxe^{4x} + 8Ce^{4x} - 6Ce^{4x} - 24Cxe^{4x} + 8Cxe^{4x} = 8e^{4x}$

$8Cx + 8C - 6C - 24Cx + 8Cx = 8$

$2C = 8$

$C = 4 \quad \therefore y_{P.I} \Rightarrow y = 4xe^{4x}$

$\therefore$ G.S

$y = Ae^{4x} + Be^{2x} + 4xe^{4x}$

(6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

$m^2 + 4m + 5 = 0$

$a=1 \quad b=4 \quad c=5$

$\frac{-4 \pm \sqrt{4^2 - 4ac}}{2a}$

$\frac{-4 \pm \sqrt{16 - 20}}{2}$

$\frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2i}{2} = -2 \pm i$

$m = -2 \pm i$

$y = e^{-2x}(A\cos x + B\sin x)$

$y = Ce^{-2x}$

$\frac{dy}{dx} = C(-2e^{-2x}) + Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x} + Ce^{-2x}$

$\frac{d^2y}{dx^2} = -2C(-2e^{-2x}) + e^{-2x}(-1) + 2Ce^{-2x}$

$= 4Ce^{-2x} - 2Ce^{-2x} - 2Ce^{-2x}$

$= 4Ce^{-2x} - 4Ce^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + 4(-2Ce^{-2x} + Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 4Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$4C - 8C + 4C + 5C = 2$

$C = 2$

$y = 2e^{-2x}$

at $x=0 \quad y=1 \quad \frac{dy}{dx} = -2$

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

$x=0, y=1$

$1 = e^{0}(A\cos 0 + B\sin 0) + 2e^{0}$

$1 = A + 2 \quad A = -1$

$\frac{dy}{dx} = e^{-2x}(-A\sin x + B\cos x) + (A\cos x + B\sin x)(-2e^{-2x}) + 4e^{-2x}$

$x=0 \quad \frac{dy}{dx} = -2$

$-2 = B + A(-2) + 4$

$-2 = B - 2A - 4$

$-2A = B - 2$

$2 = B - 2A$

$B = 2 + 2A = 2 + 2(-1)$

$= 2 - 2 = 0$

$B = 0$

$y = e^{-2x}(-\cos x + 0) + 2e^{-2x}$

$y = e^{-2x}(2 - \cos x)$

(5) $\frac{d^2y}{dx^2} - \frac{2dy}{dx} + y = x^2 \sin x$

$m^2 - 2m + 1 = 0$

$m = 2 \pm \sqrt{4-4}$

$m = 1$

$y = e^x (A + Bx)$

$y = x \sin 2x$

$y = (\cos x + B \sin x) x$

$\frac{dy}{dx} = -C \sin x + B \cos x$

$\frac{d^2y}{dx^2} = -(C \cos x + B \sin x)$

$-(C \cos x + B \sin x) - 2(-C \sin x + B \cos x) + (C \cos x + B \sin x) x = x \sin x$

$-C \cos x - B \sin x + 2C \sin x - 2B \cos x + C x \cos x + B x \sin x = x \sin x$

$-C \cos x - 2B \cos x + C x \cos x - B \sin x + 2C \sin x + B x \sin x = x \sin x$

$\cos x (-C - 2B + Cx) + \sin x (-B + 2C + Bx) = x \sin x$

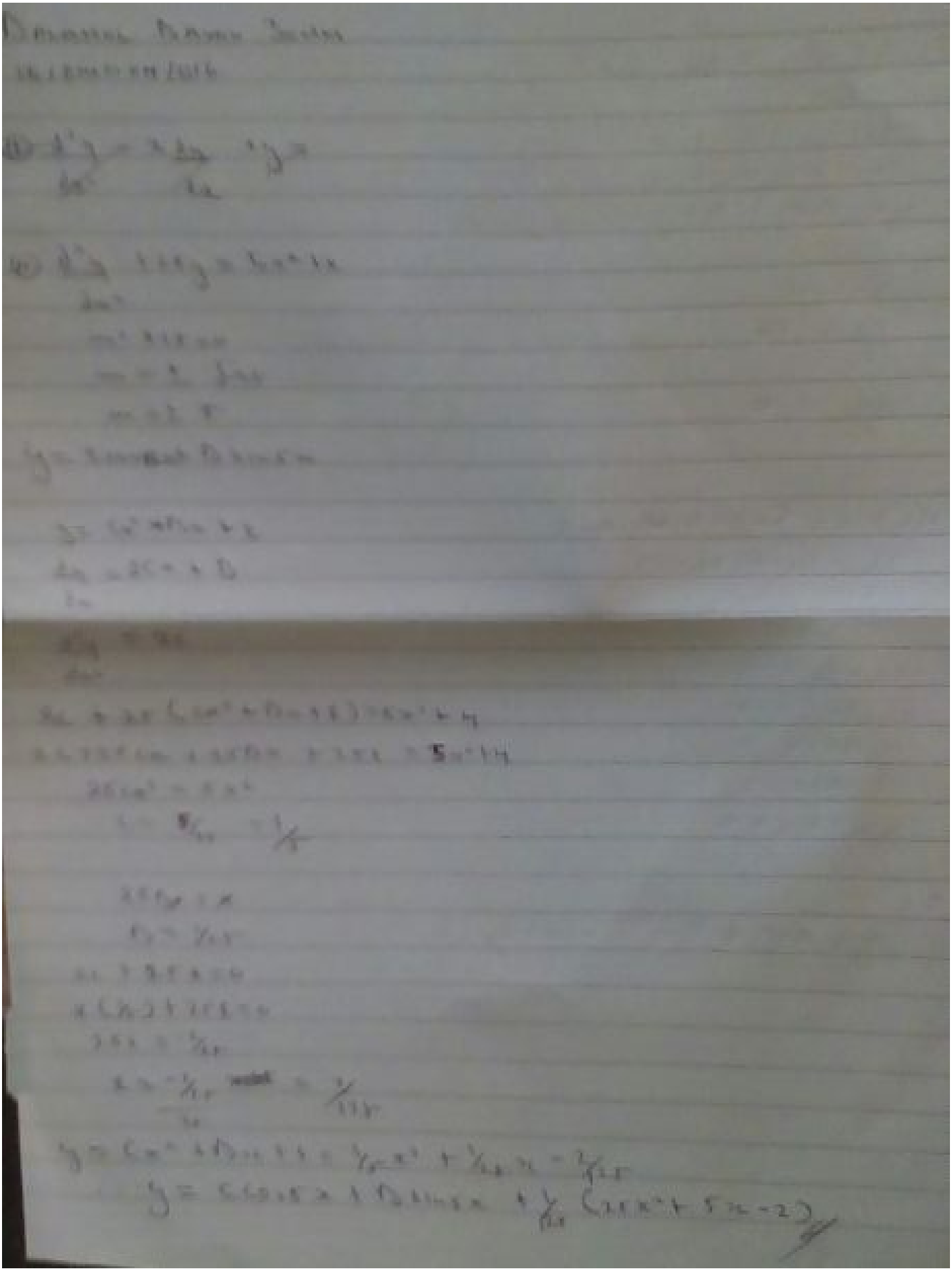
$\cos x (-2B) + \sin x (2C) = x \sin x$

$-2B = 0 \quad \therefore B = 0$

$2C = 1 \quad C = \frac{1}{2}$

P.I $y = \frac{1}{2} x \cos x$

$\therefore$ G.S $y = e^x (A + Bx) + \frac{1}{2} x \cos x$



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(3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 1y = e^{-2x}$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$m_1 = -1 \quad m_2 = -1$

$y = e^{-x}(A + Bx)$.

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4C - 4C + C = 1$

$\therefore C = 1$

$y = e^{-2x}$

G.S $\Rightarrow y = e^{-x}(A + Bx) + e^{-2x}$

Congratulations on your Retirement

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Q $\frac{d^2y}{dx^2} - 4y = 10e^{2x}$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm\sqrt{4}$

$m = \pm 2$

$y = Ae^{2x} + Be^{-2x}$

$y = Ce^{2x}$

$\frac{dy}{dx} = 3Ce^{2x}$

$\frac{d^2y}{dx^2} = 9Ce^{2x}$

$9Ce^{2x} - 4Ce^{2x} = 10e^{2x}$

$9C - 4C = 10$

$5C = 10$

$C = 2$

$P_I \Rightarrow y = 2e^{2x}$

$GS \Rightarrow y = Ae^{2x} + Be^{-2x} + 2e^{2x}$

[illegible]

[illegible]

① $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Forming auxiliary equation

$m^2 - m - 2 = 0$

$m^2 - m + 2m = 0$

$m(m-1) + 1(m-1)$

$m_1 = 1 \quad m_2 = 2$

$\therefore$ CF $y = Ae^{x} + Be^{2x}$

PI $y = c$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

$\therefore \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$0 - 0 - 2(c) = 8$

$-2c = 8$

$c = \frac{8}{-2} = -4$

G.S $\Rightarrow$ $y = Ae^{x} + Be^{2x} - 4$