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16/ENG04/032
ELECT/ELECT
ENG 281

ASSIGNMENT

The parametric equ of a curve are as given as equ

$x = \cos t + t\sin t$ — ①

$y = \sin t - t\cos t$ — ②

In terms of t, determine

a) an expression for the radius of curvature (R)

$$\Rightarrow R = \frac{\left[1 + \left[\frac{dy}{dx}\right]^2\right]^{3/2}}{d^2y/dx^2}$$

$x = \cos t + t\sin t$

$dx/dt = t\cos t$

$y = \sin t - t\cos t$

$dy/dt = \cos t - (-t\sin t + \cos t)$

$= \cos t + t\sin t - \cos t$

$= t\sin t$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= t\sin t \times \frac{1}{t\cos t} = \frac{t\sin t}{t\cos t} = \frac{\sin t}{\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dt}\left(\frac{dy}{dx}\right) \times \frac{dt}{dx}$$

$$= \frac{d}{dt}\left(\frac{\sin t}{\cos t}\right) \times \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{V\,{}^{dy}/_{dx} - U\,{}^{dv}/_{dx}}{V^2}$$

where $u = \cos t$

$dy = \sin t$

$u = \sin t$

$$\frac{d^2y}{dx^2} = \frac{\cos^2 t \ \sin^2 t}{\cos^2 t} \times \frac{dt}{dx}$$

Recall ; $\cos^2 t + \sin^2 t = 1$

$$\frac{1}{\cos^2 t} \times \frac{1}{t \cos t} = \frac{1}{t \cos^3 t}$$

Also recall that

$$R = \frac{\left[1 + \left[{}^{dy}/_{dx}\right]^2\right]^{3/2}}{{}^{d^2y}/_{dx^2}}$$

$$R = \frac{\left[1 + \left({}^{\sin t}/_{\cos t}\right)^2\right]^{3/2}}{{}^{1}/_{\cos^3 t}}$$

$$R = \left(1 + {}^{\sin 2t}/_{\cos^2 t}\right)^{3/2} \times {}^{t \cos^3 t}/_{1}$$

$$R = \left[(\cos^2 t + \sin^2 t)/\cos^2 t\right] \times t \cos^3 t$$

$$R = \frac{1}{(\cos^3 t)^{3/2}} \times t \cos^3 t = \frac{1}{(\sqrt{\cos^2 t})^3} \times t \cos^3 t$$

$$R = \frac{t \cancel{\cos^3 t}}{\cancel{\cos^3 t}} = t$$

$\therefore$ The radius of curvature is t

b) Expressions for the co-ordinates (h, k) of the Centre curvature :

$\Rightarrow h = x_1 - R \sin \theta$ ①

$k = y_1 + R \cos \theta$ ②

$R = tt = \theta$

$x_1 = \cos t + t \sin t$

$y_1 = \sin t + t \cos t$

Substituting x_1 in equ ① and y in equ ②

$h = \cos t + t\sin t - t\sin t$

$h = \cos t$

$k = \sin t - t\cos t + t\cos t$

$k = \sin t$

The expressions for the co-ordinates (h, k) of the of Curvature is $(\cos t, \sin t)$.