$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 6\sin\theta$$

Solution

$m^2 + 4m + 5 = 0$

Using $m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-4 \pm \sqrt{4 \cdot i}}{2}$

$m = \frac{-4}{2} \pm \frac{2i}{2}$

$m = 2 \pm i$

$y = e^{2x}(\cos\theta + i\sin\theta)$

P.I

$y = A\cos\theta + B\sin\theta$

$\frac{dy}{d\theta} = -A\sin\theta + B\cos\theta$

$\frac{d^2y}{d^2\theta} = -A\cos\theta - B\sin\theta$

Comparing coefficient

$-A + 4B + 5A = 0$

$4A + 4B = 0$ —— eqn 1)

$-B - 4A + 6B = 6$

$-4A + 4B = 6$ —— eqn (ii)

add eqn i and ii

$8B = 6$

$B = 3/4$

$4A + 4(\frac{3}{4}) = 0$

$4A = -3 \qquad A = -3/4$

$y = -\frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$ —— P.I

$y = e^{-2\theta}(C\cos\theta + D\sin\theta) - \frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$

at $\theta = \infty$ and $\frac{dy}{dx} = 0$

$\frac{dy}{d\theta} = e^{-2\theta}[-C\sin\theta + D\cos\theta] + [C\cos\theta + D\sin\theta]$
$- 2e^{-2\theta} + \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$

at $\theta = \alpha$ and $\frac{dy}{dx} = 0$

$0 = \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$

$+\frac{3}{4}\sin\theta = -\frac{3}{4}\cos\theta$

$-\cos\theta = \sin\theta$

D.B.S by $-\cos\theta$

$$\frac{-\cos\theta}{-\cos\theta} = \frac{\sin\theta}{-\cos\theta}$$

$$\frac{\sin\theta}{-\cos\theta} = 1$$

$-\tan\theta = 1$

$\tan\theta = -1$

$\theta = \tan^{-1}(1)$

$\theta = 45^0$

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