

NAME: IKHENGBA ELIZABETH
 AMANIDA
 DEPT: CIVIL ENGINEERING
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(1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$0 = 2s + 0 + m$
 $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

$m^2 - m - 2 = 0$

$m^2 + m - 2m - 2 = 0$

$m(m+1) - 2(m+1) = 0$

$m - 2 = 0, m + 1 = 0$

$m = 2, m = -1$

CF
 $y = Ae^{m_1x} + Be^{m_2x}$

$y = Ae^{2x} + Be^{-x}$

P.I = $y = c$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$0 - 0 - 2(c) = 8$

$-2c = 8$

$c = \frac{-8}{2}$

$c = -4$

GS = CF + P.I

$= Ae^{2x} + Be^{-x} - 4$

(2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$\frac{d^2y}{dx^2} - 4y = 0$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm\sqrt{4}$

$m = \pm 2$

CF

$y = A\cosh 2x + B\sinh 2x$

P.I

$y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

$\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$5C = 10$

$C = \frac{10}{5}$

$C = 2$

$\therefore y = Ce^{3x}$

$= 2e^{3x}$

$\therefore GS = CF + P.I$

$y = A\cosh 2x + B\sinh 2x + 2e^{3x}$

(B)

$$3) \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$$

$$m^2 + 2m + 1 = 0$$

$$m^2 + m + m + 1 = 0$$

$$m(m+1) + 1(m+1) = 0$$

$$(m+1)(m+1) = 0$$

$$m = -1 \text{ (twice), } m$$

$$CF: y = e^{mx} (A + Bx)$$

$$CF: y = e^{-x} (A + Bx)$$

P.I : $x^2 + x + A = 0$

$$y = Ce^{-2x}$$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4Ce^{-2x}$$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$$

$$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$$

$$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$$

$$Ce^{-2x} = e^{-2x}$$

$$C = 1$$

$$\therefore y = e^{-2x}$$

GS = CF + PI

$$y = e^{-x} (A + Bx) + e^{-2x}$$

(A)

$$4) \frac{d^2y}{dx^2} + 25y = 5x^2 + x$$

$$\frac{d^2y}{dx^2} + 25y = 0$$

$$m^2 + 25 = 0$$

$$m^2 = -25$$

$$m = \pm \sqrt{-25}$$

$$m = \pm 5i$$

$$CF: y = A \cos 5x + B \sin 5x$$

P.I

$$y = Cx^2 + Dx + E$$

$$\frac{dy}{dx} = 2Cx + D$$

$$\frac{d^2y}{dx^2} = 2C$$

$$\frac{d^2y}{dx^2} + 25y = 5x^2 + x$$

$$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$$

$$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$$

$$25Cx^2 = 5x^2$$

$$C = \frac{5}{25} = \frac{1}{5}$$

$$C = 1/5$$

$$2C + 25E = 0$$

$$2(1/5) + 25E = 0$$

$$\frac{2}{5} + 25E = 0$$

$$25E = -\frac{2}{5}$$

$$125E = -2$$

$$E = \frac{-2}{125}$$

VISTALINE

$$25Dx = x$$

$$D = \frac{x}{25}$$

$$D = \frac{1}{25}$$

$$y = Cx^2 + Dx + E$$

$$\frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$$

$$G.S = C.F + P.I$$

$$y = A \cos 5x + B \sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$$

5

$$15) \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

$$m^2 - 2m + 1 = 0$$

$$m^2 - m - m + 1 = 0$$

$$m(m-1) - 1(m-1) = 0$$

$$(m-1)(m-1) = 0$$

$$m = 1 \text{ (twice)}$$

$$C.F = y = e^x (A + Bx)$$

$$P.I \ y = C \cos x + D \sin x$$

$$\frac{dy}{dx} = -C \sin x + D \cos x$$

$$\frac{d^2y}{dx^2} = -C \cos x - D \sin x$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$$

$$-C \cos x - D \sin x - 2(-C \sin x + D \cos x) + C \cos x + D \sin x = 4\sin x$$

$$-C \cos x - D \sin x + 2C \sin x - 2D \cos x + C \cos x + D \sin x = 4\sin x$$

$$\cos x (-C - 2D + C) + \sin x (-D + 2C + D) = 4\sin x$$

$$-2D = 0$$

$$D = 0/2 = 0$$

$$2C = 4$$

$$C = 4/2$$

$$C = 2$$

$$P.I = y = 2 \cos x + 0 \sin x$$

$$y = 2 \cos x$$

$$G.S \Rightarrow C.F + P.I$$

$$y = e^{2x} (A + Bx) + 2 \cos x$$

6

$$6. \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$$

Given that $x=0, y=1$ & $\frac{dy}{dx} = -2$

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$$

$$m^2 + 4m + 5 = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$$

$$\frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$\frac{-4 \pm \sqrt{-4}}{2}$$

$$\frac{-4 \pm j2}{2} = -2 \pm j$$

$$y = e^{-2x}(A \cos x + B \sin x)$$

P.I;

$$y = Ce^{-2x}$$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4Ce^{-2x}$$

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$$

$$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$$

$$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$$

$$Ce^{-2x} = 2e^{-2x}$$

$$C = 2$$

$$P.I = y = Ce^{-2x}$$

$$y = 2e^{-2x}$$

$$\therefore y = e^{-2x}(A \cos x + B \sin x) + 2e^{-2x}$$

at $x=0, y=1$

$$1 = e^{-2(0)}(A \cos(0) + B \sin(0)) + 2e^{-2(0)}$$

$$1 = 1(A+0) + 2$$

$$1 = A + 2$$

$$1 - 2 = A$$

$$-1 = A$$

$$A = -1$$

at $x=0, \frac{dy}{dx} = -2$

$$\frac{dy}{dx} = [-2e^{-2x}(A \cos x + B \sin x) + e^{-2x}(-A \sin x + B \cos x)] - 4e^{-2x}$$

$$-2 = [-2e^{-2(0)}(A \cos 0 + B \sin 0) + e^{-2(0)}(-A \sin 0 + B \cos 0)] - 4e^{-2(0)}$$

$$-2 = [-2(A+0) + 1(0+B) - 4]$$

$$A = -1$$

$$-2 = [(-2(-1) + B) - 4]$$

$$-2 = [2 + B - 4]$$

$$-2 = 2 - 4 + B$$

$$-2 = -2 + B$$

$$B = -2 + 2 = 0$$

$$A = -1, B = 0$$

$$G.S = y = e^{-2x}(-1 \cos x + 0 \sin x) + 2e^{-2x}$$

$$y = e^{-2x}(-\cos x) + 2e^{-2x}$$

$$y = e^{-2x}(2 - \cos x)$$

7

$$7) \frac{3d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$$

$$\frac{3d^2y}{dx^2} - 2\frac{dy}{dx} - y = 0$$

$$3m^2 - 2m - 1 = 0$$

$$3m^2 - 3m + m - 1 = 0$$

$$3m(m-1) + 1(m-1) = 0$$

$$(3m+1)(m-1) = 0, (m-1) = 0$$

$$3m = -1, m = 1$$

$$m = -1/3; m = 1$$

$$\therefore C.F. = y = Ae^{mx} + Be^{nx} \\ = Ae^{-1/3x} + Be^x$$

$$P.I. y = Cx + D$$

$$\frac{dy}{dx} = C$$

$$\frac{d^2y}{dx^2} = 0$$

$$\frac{3d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$$

$$3(0) - 2(C) - (Cx + D) = 2x - 3$$

$$-2C - Cx - D = 2x - 3$$

$$-Cx - Cx = 2x$$

$$C = -2$$

$$-2C - D = -3$$

$$-2(-2) - D = -3$$

$$4 - D = -3$$

$$-D = -3 - 4$$

$$+D = +7 \therefore D = 7$$

$$\therefore y = -2x + 7$$

$$y = -2x + 7$$

$$G.S. \Rightarrow C.F. + P.I.$$

$$y = Ae^{-1/3x} + Be^x - 2x + 7$$

8

$$8) \frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$$

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$$

$$m^2 - 6m + 8 = 0$$

$$m^2 - 2m - 4m + 8 = 0$$

$$m(m-2) - 4(m-2) = 0$$

$$(m-4)(m-2) = 0, (m-2) = 0$$

$$m = 4, m = 2$$

$$y = Ae^{mx} + Be^{nx}$$

$$y = Ae^{4x} + Be^{2x}$$

$$P.I. y = Ge^{4x}$$

$$\frac{dy}{dx} = 4Ge^{4x} + Ce^{4x}$$

$$\frac{d^2y}{dx^2} = 16Ge^{4x} + 4Ce^{4x} + 4Ce^{4x}$$

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$$

$$(16Ge^{4x} + 4Ce^{4x} + 4Ce^{4x} - 6(4Ge^{4x} + Ce^{4x}) + 8(Ge^{4x} + Ce^{4x}))$$

$$+ 8(Ce^{4x}) = 8e^{4x}$$

$$(16Ge^{4x} - 24Ge^{4x} + 8Ge^{4x} + 4Ce^{4x} + 4Ce^{4x} - 6Ce^{4x} - 6Ce^{4x} + 8Ce^{4x} + 8Ce^{4x}) = 8e^{4x}$$

$$4Ce^{4x} + 4Ce^{4x} - 6Ce^{4x} = 8e^{4x}$$

$$8Ce^{4x} - 6Ce^{4x} = 8e^{4x}$$

$$2Ce^{4x} = 8e^{4x}$$

$$2C = 8$$

$$C = \frac{8}{2}$$

$$C = 4$$

$$P.I. = 4xe^{4x}$$

$$G.S. = Ae^{4x} + Be^{4x} + 4xe^{4x}$$