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16/ENG02/050
Computer Engineering
Eng 281 (Assignment)
200 level

1. Evaluate the following limits of function,

a) $$\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2} \right]$$

$$= \lim \left[\frac{\frac{d}{dx} (x^2 - \pi/4)\sin(\cos x)}{d/dx\,(x - \pi/2)} \right]$$

$$\frac{d}{dx}\left[(x^2 - \pi/4)\sin(\cos x)\right] = \frac{udu}{dx} + \frac{vdu}{dx}$$

let $u = x^2 - \pi/4$

$$\frac{du}{dx} = 2x$$

Let $v = \sin(\cos x)$

let $w = \cos x$

$$\frac{dv}{dw} = \cos w$$

$$\frac{dw}{dx} = \cos x$$

$$\frac{dw}{dx} = -\sin x$$

$$\frac{dv}{dx} = \frac{dv}{dw} \cdot \frac{dw}{dx}$$

$$= \cos w(-\sin x)$$

$$\frac{dv}{dx} = -\sin x \cos(\cos x)$$

$$\frac{d}{dx}\left[(x^2 - \tfrac{\pi}{4})\sin(\cos x)\right]$$

$$= (x^2 - \tfrac{\pi}{4})(-\sin x \cos(\cos x))$$

$$+$$

$$\sin(\cos x)(2x)$$

$$\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2} \right]$$

$$= \lim_{x \to \pi/2} \left[\frac{(x^2 - \frac{\pi}{4})(-\sin x \cos(\cos x)) + 2x\sin(\cos x)}{x} \right]$$

$$= \frac{\left[(\frac{\pi}{2})^2 - \frac{\pi}{4}\right]\left[-\sin\frac{\pi}{2}\cos(\cos\frac{\pi}{2})\right] + \left[2(\frac{\pi}{2})\sin(\cos\frac{\pi}{2})\right]}{\pi/2}$$

$$= \frac{\left[\frac{\pi^2}{4} - \frac{\pi}{4}\right]\left[-1 \times \cos(0)\right] + \left[\pi \sin(0)\right]}{\pi/2}$$

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$$= \left[\frac{\pi^2 - \pi}{4}\right]\left[-1\right]_{\pi/2}^{\pi/2} + 0$$

$$= \frac{-\pi^2 + \pi}{4} - \frac{\pi}{2}$$

$$= \frac{-\pi^2 + \pi}{\cancel{4}\,2} \times \frac{\cancel{2}\,1}{\pi}$$

$$= \frac{\pi(-\pi + 1)}{2} \times \frac{1}{\pi}$$

$$= \frac{1 - \pi}{2}$$

1b $\lim_{x \to \pi/2} \ln\left[e\left(\frac{3x^2+2x-1}{x+1}\right)\right]$

$$= \lim_{x \to \pi/2}\left[\frac{3x^2+2x-1}{x+1}\right]$$

$$= \lim_{x \to \pi/2} \frac{(3x-1)\cancel{(x+1)}}{\cancel{(x+1)}}$$

$$= \lim_{x \to \pi/2} (3x-1)$$

$$= 3\left(\frac{\pi}{2}\right) - 1$$

$$= 3\frac{\pi}{2} - 1$$

1c $\lim_{x \to 2+\sqrt{3}} \left[\cos\left(\sin^{-1}\left(\frac{(x-2)}{(x-\sqrt{3})}\right)\right)\right]$

$$= \cos\left[\sin^{-1}\left[\frac{2+\sqrt{3}-2}{2+\sqrt{3}-\sqrt{3}}\right]\right]$$

$$= \cos\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right] = \cos 60 = \frac{1}{2}$$

1d) $\lim_{x \to 4}\left(\frac{x^2-8x+16}{x^2-5x+4}\right)$

$$= \lim_{x \to 4}\left(\frac{(x-4)\cancel{(x-4)}}{(x-1)\cancel{(x-4)}}\right)$$

$$= \lim_{x \to 4}\left[\frac{x-4}{x-1}\right]$$

$$= \frac{4-4}{4-1} = 0$$

2a $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \cdots$

$U_n = \frac{2}{(n+1)(n+2)}$ $\quad U_n+1 = \frac{2}{(n+2)(n+3)}$

$\frac{U_n+1}{U_n} = \frac{2}{(n+2)(n+3)} \times \frac{(n+1)(n+2)}{2}$

$= \frac{n+1}{n+3}$

$\lim_{n\to\infty} \frac{U_n+1}{U_n} = \lim_{n\to\infty} \frac{n+1}{n+3}$

$= \frac{\frac{n}{n} + \frac{1}{n}}{\frac{n}{n} + \frac{3}{n}} = \frac{1+\frac{1}{n}}{1+\frac{3}{n}} = \frac{1+0}{1+0} = \frac{1}{1} = 1$

Since $\lim_{n\to\infty} \frac{U_n+1}{U_n+1}$

The serie is ~~in~~ conclusive

2b. $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$

$U_n = \frac{2}{n^2}$ $\quad U_n+1 = \frac{2}{(n+1)^2}$

$\lim_{n\to\infty} = \frac{U_n+1}{U_n} = \frac{2}{(n+1)^2} = n^2$

$= \frac{n^2}{(n+1)^2} = \frac{n^2}{n^2+2n+1}$

$\lim_{n\to\infty} = \frac{\frac{n^2}{n^2}}{\frac{n^2}{n^2} + \frac{2n}{n^2} + \frac{1}{n^2}} = \frac{1}{1+\frac{2}{n}+\frac{1}{n^2}} = \frac{1}{1} = 1$

2. $\lim_{n\to\infty} U_n = \lim_{n\to\infty} = \frac{2}{n^2}$

$\lim_{n\to\infty} = \frac{2/n^2}{n^2/n^2} = \frac{0}{1} = 0$

∴ The Series is convergent

2c. $U_n = \frac{1 + 2n^2}{1 + n^2}$

$\lim_{n\to\infty} = \frac{1 + 2n^2}{1 + n^2} = \frac{\frac{1}{n^2} + \frac{2n^2}{n^2}}{\frac{1}{n^2} + n^2/n^2}$

$= \frac{0+2}{1} = 2$

The series is divergent

3 $\frac{x}{127} + \frac{x^2}{125} + \dots + \frac{x^n}{(2+1)^2}$

$U_n = \frac{x^n}{(2n+1)^3} = U_{n+1} = \frac{x^{n+1}}{(2n+2)^2}$

$\lim \frac{U_{n+1}}{U_n} = \frac{x^{n+1}}{(2n+2)^3} \times \frac{(2n+1)^3}{x^n}$

$\frac{x(2n+1)^3}{(2n+2)^3} = \frac{8n^3 + 12n^2 + n + 1}{8n^3 + 24n^2 + 24n + 8}$

Divide by n^3

$= \frac{(8 + \frac{2}{n} + \frac{1}{n^2} + \frac{1}{n^3})}{8 + 12/n + 24/n^2 + 8/n^3)}$

as $n \to \infty$, $1/n \to 0$

$\frac{8x}{8} \geq x - 1, \; x < 1$

4. Evaluate using L'Hopitalls Rule

$\lim_{x\to 0} \left(\frac{\sin x - \cos x}{x^3} \right)$

$= \lim_{x\to 0} \left(\frac{\cos x + \sin x}{3x^2} \right)$

$\lim_{x\to 0} \left(\frac{-\sin x + \cos x}{6x} \right)$

$= \frac{-\cos 0 - \sin 0}{6}$

$= \frac{-1 - 0}{6}$

$= \frac{-1}{6}$