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The parameter equation of the curve os given

$x = \cos t + t\sin t$

$y = \sin t - \cos t$

To find the radius of curvature [R] in terms of t

$\frac{dx}{dt} = -\sin t + [t\cos t + \sin t]$

$= -\sin t + \cos t + \sin t$

$\frac{dx}{dt} = t\cos t$

$\frac{dy}{dt} = t\sin t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{t\sin t}{1} \times \frac{1}{t\cos t}$

$dy/dx = t\sin t / t\cos t$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[dy/dx\right] = \frac{d}{dt}\left[\frac{t\sin t}{t\cos t}\right]\frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{[t\cos t][1\cos t + \sin t] - [t\sin t][-t\sin t + t\cos t]}{[t\cos t]^2} \times \frac{dt}{dx}$$

$$= \frac{t^2\cos^2 t + t\cos t\sin t + t^2\sin^2 t - t\cos t\sin t}{[t\cos t]^2} \times \frac{1}{t\cos t}$$

$$= \frac{t^2\cos^2 t + t^2\sin^2 t}{[t\cos t]^2} \times \frac{1}{t\cos t}$$

$$= \frac{t^2[\cos^2 t + \sin^2 t]}{t^2\cos^2 t} \times \frac{1}{t\cos t}$$

Recall $\cos^2 t + \sin^2 t = 1$

$$= \frac{1}{t^2\cos^2 t} \times \frac{1}{t\cos t}$$

$$\therefore R = \frac{\left[1 + \left[\frac{dy}{dx}\right]^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$= \frac{\left[1 + \left[\frac{t\sin t}{t\cos t}\right]^2\right]^{3/2}}{1/t\cos^3 t}$$

$$= \left[\frac{1}{1} + t^2\sin^2 t\right]^{3/2} \times \frac{t\cos^3 t}{1}$$

$$= \left[\frac{t^2 + 1}{t^2\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$= \left[\frac{1}{\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$= \left[\frac{\sqrt{1}}{\sqrt{\cos t}}\right]^3 \times t\cos^3 t$$

$$= \frac{1^3}{\cos^3 t} \times \frac{t\cos^3 t}{1}$$

$R = t$

Therefore, the radius of curvature in terms of t is $R = t$

ii) for the coordinates of the centre of curvature $[h, k]$

$h = x_1 - R\sin\theta$

$k = y_1 + R\cos t$

$\tan\theta = dy/dx = t\sin t / t\cos t = \tan t$

$\theta = \tan^{-1}[\tan t]$

$\theta = t$

$x_1 = \cos t + t\sin t$ from eqn [i]

$y_1 = \sin t - t\cos t$ from eqn [ii]

$h = \cos t + t\sin t - [t][\sin t]$

$h = \cos t + t\sin t - t\sin t$ $\quad h = \cos t$

$k = \sin t - t\cos t + [t][\cos t]$

$k = \sin t - t\cos t + t\cos t$

$k = \sin t$

$h = \cos t$ and $k = \sin t$ //