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15/ENG02/041
Computer Engineering

Engineering Mathematics Assignment
ENG 381

1) The model of a system is given as in equation.

$$\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$$

i) Obtain an expression for y as a function of θ

$$\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$$

assume $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 0$

$$m^2 + 4m + 5 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{-4 \pm \sqrt{-4}}{2}$$

$$= \frac{-4 \pm 2j}{2}$$

$$= -2 \pm j$$

$$C.F = e^{-2\theta}(A\cos\theta + B\sin\theta)$$

since $f(x) = 6\sin\theta$, $y = C\cos\theta + D\sin\theta$

$$\frac{dy}{dx} = -C\sin\theta + D\cos\theta$$

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$$\frac{d^2y}{dx^2} = -C\cos\theta - D\sin\theta$$

$$= -C\cos\theta - D\sin\theta + 4(-C\sin\theta + D\cos\theta) + 5(C\cos\theta + D\sin\theta) = 6\sin\theta$$

$$= -C\cos\theta - D\sin\theta + 4D\cos\theta - 4C\sin\theta + 5C\cos\theta + 5D\sin\theta = 6\sin\theta$$

$$= (-C + 4D + 5C)\cos\theta + (-D - 4C + 5D)\sin\theta = 6\sin\theta$$

$$-C + 4D + 5C = 0 \quad \text{--- (i)}$$
$$-D - 4C + 5D = 6 \quad \text{--- (ii)}$$

$$4D + 4C = 0 \quad \text{---}$$
$$4D = -4C$$
$$D = -C \quad \text{--- (iii)}$$
$$-(-C) - 4C + 5(-C) = 6$$
$$C - 4C - 5C = 6$$
$$-8C = 6$$
$$-C = 6/8$$
$$C = -3/4$$

$$D = -C$$
$$\therefore D = +3/4$$
$$P.I = -3/4\cos\theta + 3/4\sin\theta$$

general solution
$$y = e^{-2\theta}(A\cos\theta + B\sin\theta) - 3/4\cos\theta + 3/4\sin\theta$$

(iii) Using the expression obtained above

(i) estimate the value of θ at steady state

at steady state $\theta = \infty$ and $dy/d\theta = 0$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}\left(e^{-2\theta}(A\cos\theta + B\sin\theta)\right) + \frac{d}{d\theta}\left(-3/4\cos\theta\right) + \frac{d}{d\theta}\left(3/4\sin\theta\right)$$

1 let $u = e^{-2\theta}$ $\quad du/d\theta = -2e^{-2\theta}$

$v = A\cos\theta + B\sin\theta$ $\quad dv/d\theta = -A\sin\theta + B\cos\theta$

$$\frac{dy}{d\theta} = v\,du/d\theta + u\,dv/d\theta$$

$$= (A\cos\theta + B\sin\theta) \cdot -2e^{-2\theta} + e^{-2\theta}(-A\sin\theta + B\cos\theta)$$

$$= -2e^{-2\theta}(A\cos\theta + B\sin\theta) + e^{-2\theta}(-A\sin\theta + B\cos\theta)$$

$$\therefore \frac{dy}{d\theta} = -2e^{-2\theta}(A\cos\theta + B\sin\theta) + e^{-2\theta}(-A\sin\theta + B\cos\theta) + \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$$

$\theta = \infty \qquad dy/d\theta = 0$

$$\therefore \quad 0 = -2e^{-2\infty}(A\cos\theta + B\sin\theta) + e^{-2\infty}(-A\sin\theta + B\cos\theta) + \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$$

$e^{-\infty} = 0$

$$\therefore 0 = -2(0)(A\cos\theta + B\sin\theta) + 0(-A\sin\theta + B\cos\theta) + 3/4\sin\theta + \frac{3}{4}\cos\theta$$

$$0 = 3/4\sin\theta + 3/4\cos\theta$$

$$3/4(\sin\theta + \cos\theta) = 0$$

(2) The equation of bending for a horizontal cantilever having length l with load w per unit length is given in the equation as

$$EI\frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2$$

where E, I, w and L are constants. If $y=0$ and $\frac{dy}{dx}=0$ at $x=0$, using the Auxilary equation method. find y as a function of x. Hence, evaluate the value of y when $x=L$.

$$EI\frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2$$

$$\frac{d^2y}{dx^2} = \frac{w}{2EI}(L-x)^2$$

let $\frac{w}{2EI} = A_0$

$$\frac{d^2y}{dx^2} = A(L-x)^2$$

$$\frac{d^2y}{dx^2} = A(L^2 - 2xL + x^2)$$

$$\frac{d^2y}{dx^2} = AL^2 - 2AxL + Ax^2$$

assume $\frac{d^2y}{dx^2} = 0$

$m^2 = 0$

$m = \sqrt{0}$

$m = 0$ twice

$\therefore$ C.F $= e^{0x}(A+Bx)$

$= A + Bx$

since $f(x) = A_0L^2 - 2A_0xL + A_0x^2$, $y = Cx^4 + Dx^3 + Ex^2$

$$\frac{dy}{dx} = 4Cx^3 + 3Dx^2 + 2Ex$$

$$\frac{d^2y}{dx^2} = 12Cx^2 + 6Dx + 2E$$

$$\therefore 12Cx^2 + 6Dx + 2E = A_0L^2 - 2A_0xL + A_0x^2$$

$$12C = A_0 \qquad 6D = -2A_0L \qquad 2E = A_0L^2$$

$$12C = \frac{w}{2EI} \qquad 6D = \frac{-2wL}{2EI} \qquad 2E = \frac{wL^2}{2EI}$$

$$C = \frac{w}{24EI} \qquad D = \frac{-wL}{6EI} \qquad E = \frac{wL^2}{4EI}$$

$$\therefore P.I = \left(\frac{w}{24EI}\right)x^4 + \left(\frac{-wL}{6EI}\right)x^3 + \left(\frac{wL^2}{4EI}\right)x^2$$

$$P.I = \left(\frac{w}{24EI}\right)x^4 - \left(\frac{wL}{6EI}\right)x^3 + \left(\frac{wL^2}{4EI}\right)x^2$$

general solution

$$y = A + Bx + \left(\frac{w}{24EI}\right)x^4 - \left(\frac{wL}{6EI}\right)x^3 + \left(\frac{wL^2}{4EI}\right)x^2$$

$$y = A + Bx + \frac{wx^4 - 4wLx^3 + 6wL^2x^2}{24EI}$$

$$y = A + Bx + \frac{W}{24EI}\left(x^4 - 4Lx^3 + 6L^2x^2\right)$$

at $x = 0 \quad y = 0$

$$0 = A + B(0) + \frac{W}{24EI}\left(0^4 - 4L(0)^3 + 6L^2(0)\right)$$

$$0 = A + 0 + \frac{W}{24EI}(0 - 0 + 0)$$

$$0 = A + 0 + 0$$

$$A = 0$$

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at $x = 0$ $\quad dy/dx = 0$

$$\frac{dy}{dx} = B + \frac{W}{24EI}\left(4x^3 - 12Lx^2 + 12L^2x\right)$$

$$0 = B + \frac{W}{24EI}\left(0 - 0 + 0\right)$$

$$0 = B + 0$$

$$B = 0$$

$$\therefore y = \frac{W}{24EI}\left(x^4 - 4Lx^3 + 6L^2x^2\right)$$

Evaluate the value of y when $x = L$

$$y = \frac{W}{24EI}\left(L^4 - 4L^4 + 6L^4\right)$$

$$y = \frac{W}{24EI}\left(3L^4\right)$$

$$y = \frac{3L^4W}{24EI}$$

$$y = \frac{\cancel{3WL^4}}{\cancel{24EI}} \qquad y = \frac{WL^4}{8EI}$$

$$y = \frac{WL^4}{8EI}$$

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